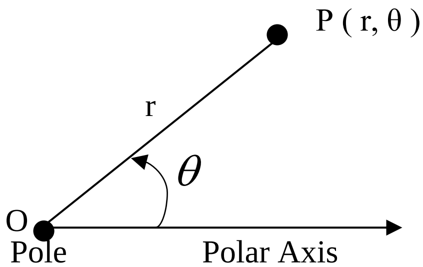
A **polar coordinate system** is a plane with a point O, called the <u>Pole</u>, and a ray from O, called the <u>Polar axis</u>. Each point P in the plane is assigned <u>r</u> <u>θ</u> as follows:

- r, is the <u>directed distance</u> from O to P
- θ , is the <u>directed Angle</u> whose initial side is on the polar axis and whose terminal side is on the line OP.

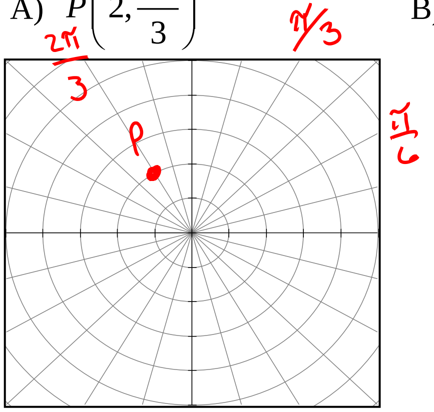


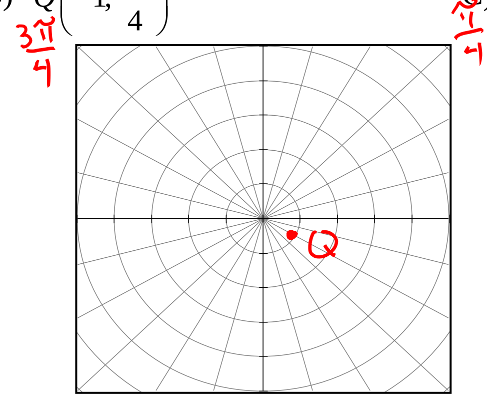
<u>EXAMPLE #1: PLOTTING POLAR POINTS</u>

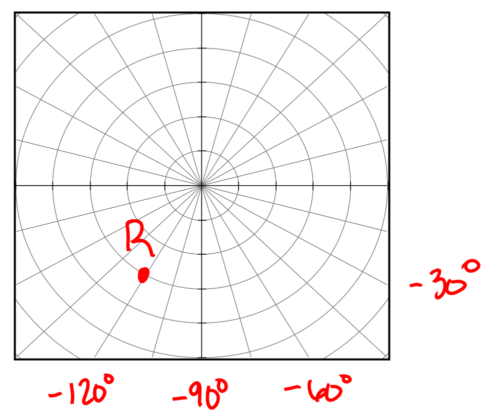
<u>Step #1</u>: Start on the Polar Axis and locate the angle. Positive is counterclockwise and negative is clockwise.
<u>Step #2</u>: If r is positive, move out that distance along the terminal side of the angle and plot the point.
<u>Step #3:</u> If r is negative, move "backwards" or in a direction opposite that of the terminal side of the angle and plot the point.

<u>Example #1:</u>
Plot the following points:

A) $P\left(2, \frac{2\pi}{3}\right)$ B) $Q\left(-1, \frac{3\pi}{4}\right)$ C) $R(3, -120°)$



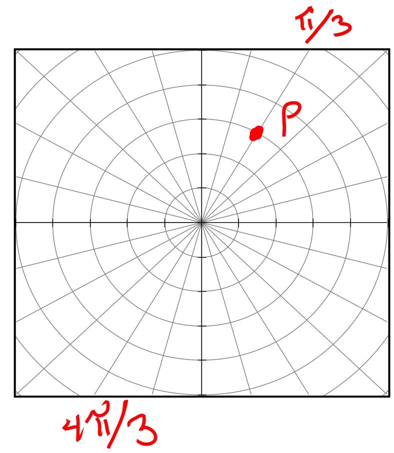




<u>EXAMPLE #2 : FINDING SEVERAL POLAR COORDINATES</u>

1st) Plot $P\left(3, \frac{\pi}{3}\right) = P\left(-3, \frac{4\pi}{3}\right) = P\left(3, -\frac{5\pi}{3}\right)$

$= P\left(3, \frac{\pi}{3} + 2\pi n\right)$

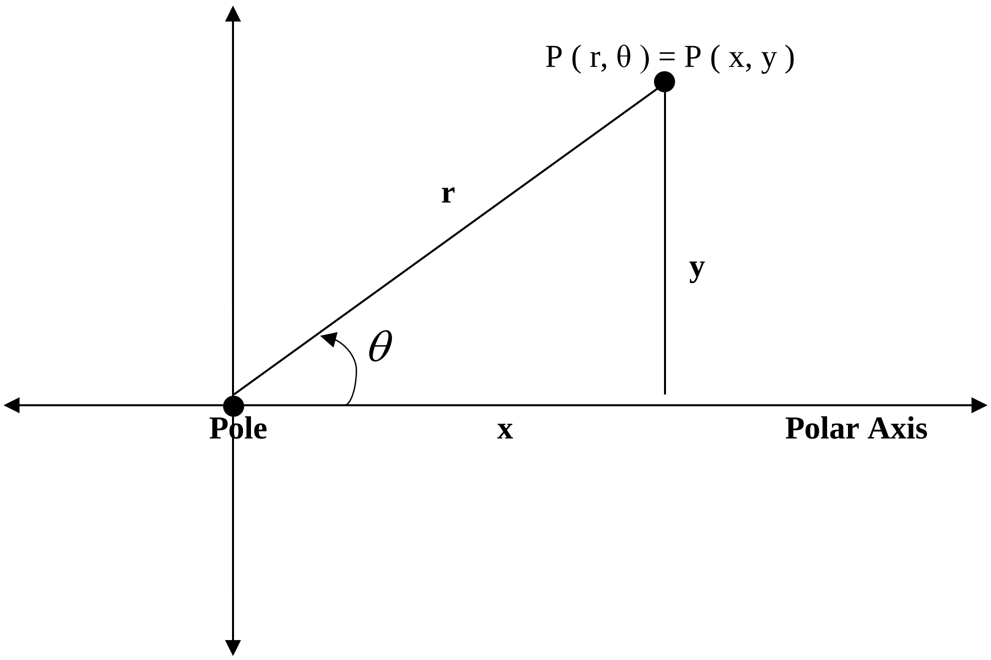


2nd) Find two additional pairs of polar coordinates for P.

$= P\left(-3, \frac{4\pi}{3} + 2\pi n\right)$

NOTE: The coordinates (r,θ) , $(r,\theta+2\pi n)$ *and* $(-r,\theta+(2n+1)\pi)$ are all name the same point. Discuss why. As a result, any point on a polar coordinate system can be specified in an infinite number of ways.

<u>COORDINATE CONVERSION</u>

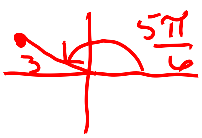


$x = r\cos\theta \qquad r^2 = x^2 + y^2$

$y = r\sin\theta \qquad \tan\theta = \dfrac{y}{x}$

<u>EXAMPLE # 3 :</u> Find the rectangular coordinates of the points with the given polar coordinates.

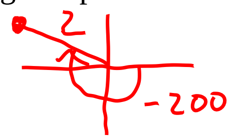
A) $P\left(3, \dfrac{5\pi}{6}\right)$



$P\left(3\cos\frac{5\pi}{6}, 3\sin\frac{5\pi}{6}\right)$

$P\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

B) $Q(2, -200°)$

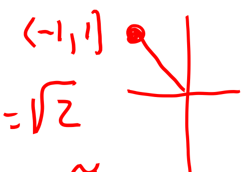


$P\left(2\cos(-200°), 2\sin(-200°)\right)$

$P(-1.879, .684)$

<u>EXAMPLE # 4 :</u> Find two polar coordinate pairs for the points with given rectangular coordinates.

A) P (-1, 1)

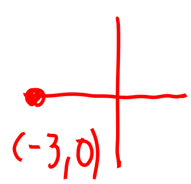


$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{-1}\right) = -\frac{\pi}{4}$

$P\left(-\sqrt{2}, -\frac{\pi}{4}\right) = P\left(\sqrt{2}, \frac{3\pi}{4}\right)$

B) Q (-3,0)



$r = \sqrt{(-3)^2 + 0^2} = 3$

$\theta = \tan^{-1}\left(\frac{0}{-3}\right) = 0°$

$Q(-3, 0°) = Q(3, 180°)$

NOTE: When converting from rectangular to polar coordinates there are <u>infinitely</u> many possible polar coordinate pairs as possible solutions because The coordinates (r,θ) , $(r,\theta+2\pi n)$ *and* $(-r,\theta+(2n+1)\pi)$ are all name the same point.