AP Calculus AB
Fundamental Thm. Practice

1. ('98AB MC#15) If $F(x) = \int_0^x \sqrt{t^3 + 1}\,dt$, the $F'(2)$=

A. -3 B. -2 C. 2 D. 3 E. 18

$F'(x) = \sqrt{x^3+1}$

$F'(2) = \sqrt{8+1} = 3$

2. ('98BC MC#91) The data for the acceleration *a(t)* of a car from 0 to 6 seconds are given in the table below.

$\int_0^6 f(x)\,dx = F(6) - F(0)$

t (sec)	0	2	4	6
a(t) (ft/sec^2)	5	2	8	3

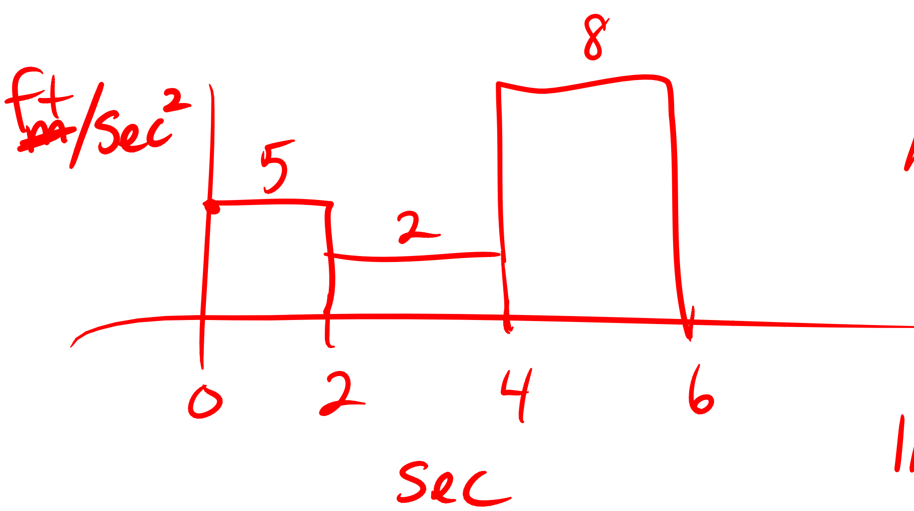
$F(0) + \underbrace{\int_0^6 f(x)\,dx}_{} = F(6)$

If the velocity at *t = 0* is 11 feet per second, the approximate value of the velocity at *t = 6*, computed using a left-hand Riemann sum with three subintervals of equal length, is

LRAM

$11 + 30 = F(6)$

A. 26 ft/sec B. 30 ft/sec C. 37 ft/sec D. 39 ft/sec E. 41 ft/sec



gain

$10 + 4 + 16 = 30$ ft/sec

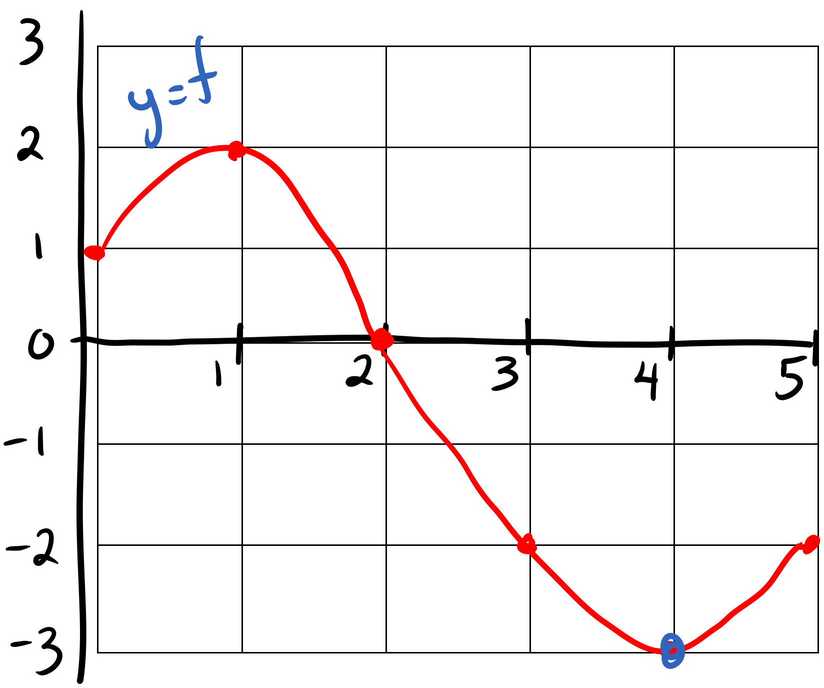
11 ft/sec + 30 ft/sec

$= \boxed{41 \text{ ft/sec}}$

$\int_a^b f(x)\,dx = F(b) - F(a)$

$F(a) + \int_a^b f(x)\,dx = F(b)$

3. ('95BC #6) The graph of f is shown below. Let $h(x)=\int_0^{\frac{x}{2}+3} f(t)\,dt$. Find $h'(2)$.



$$h'(x) = \frac{d}{dx}\int_0^{\frac{x}{2}+3} f(t)\,dt$$

$$h'(x) = \frac{1}{2} f\left(\frac{x}{2}+3\right)$$

$$h'(2) = \frac{1}{2} f\left(\frac{2}{2}+3\right)$$

$$= \frac{1}{2} \underbrace{f(4)}$$

$$= \frac{1}{2}(-3)$$

$$= -\frac{3}{2}$$

4. (from Ostebee and Zorn) The rate at which the world's oil is being consumed is increasing. Suppose that the rate (measured in billions of barrels per year) is given by the function *r(t)*, where *t* is measured in years and *t* = 0 is January1, 2010.

a. Write a definite integral that represents the total quantity of oil used between the start of 2010 and the start of 2015.

$$\int_0^5 r(t)\,dt$$

b. Suppose that $r(t)=32e^{0.05t}$. Find an approximate value for the definite integral from part (a) using a right sum with n =5 equal subintervals.

≈ 186.359 billions of barrels per year

t	y
0	32
1	33.641
2	35.365
3	37.179
4	39.085
5	41.089

c. Evaluate the definite integral from part (a) and interpret the answer.

$$\int_0^5 32e^{0.05t}\,dt = 181.78$$

5. (from Hughes-Hallet, et. al.) Suppose the density of cars, in cars/mile, for the first 30 miles from the Mass Pike from Boston during certain hours of the day can be modeled by $p(x) = 100\left(2 - \sqrt[3]{0.1x + 0.2}\right)$, where x represents the number of miles from Boston.

a. Write a function that gives the number of cars from Boston to a point x miles from Boston.

$$P(x) = \int_0^x \underbrace{100\left(2 - \sqrt[3]{0.1x + 0.2}\right)}\, dx \qquad P(x) = \int_0^x p(x)\, dx$$

b. Use this function to determine the number of cars on this thirty mile stretch of road.

$$P(30) = \int_0^{30} \left(100\left(2 - \sqrt[3]{0.1x + 0.2}\right)\right) dx \approx 2551 \text{ cars}$$

for the first 30 miles