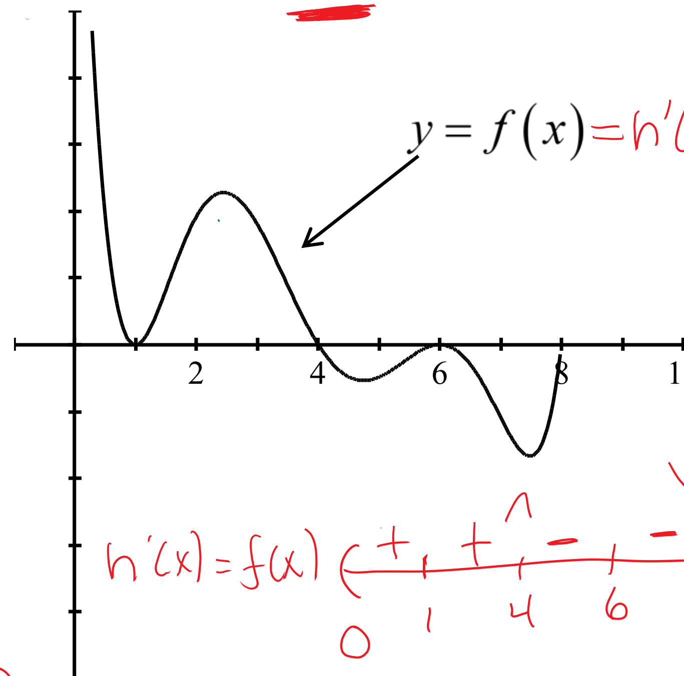
6.4 Extra Graph Practice Name Key

1. Let h be the continuous function defined by $h(x)=\int_1^x f(t)dt$ over the interval (0,8]. The graph of f(x) is given below.



$=h'(x)$

a) Find $h(1)$. $h(1)=\int_1^1 f(t)dt=0$

b) Is $h(2)$ positive or negative?
~~Justify your answer.~~

c) Is $h(0)$ positive or negative?

$h(0)=\int_1^0 f(t)dt$ negative

$h'(x)=f(x)$ (+ | + | − | −]
0 1 4 6 8

d) Find any x values where $h(x)$ has a relative max. Justify your answer.

h(x) has a relative max @ x = 4
b/c $h'(x)=f(x)$ goes from + to − @ x=4.

e) Find any x values where $h(x)$ has a relative min. Justify your answer.

h(x) has a relative min @ x=8, b/c x=8 is a local min and $h'(x)=f(x)<0$ to the left of x=8.

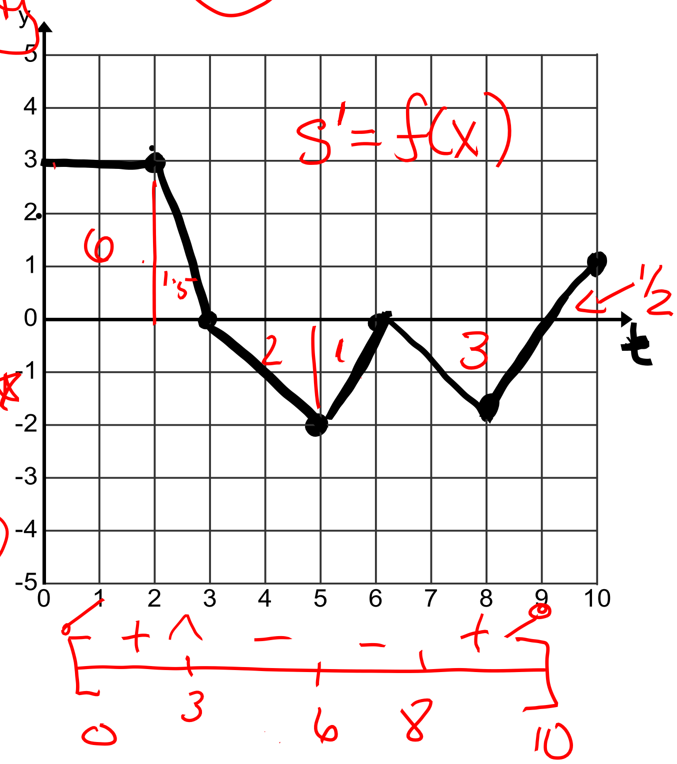
f) Find all x values of inflection points of $h(x)$

since we are given a graph of $f(x)=h'(x)$ we look when f(x) goes from increasing to decreasing or dec to inc. This implies a sign change for $h''(x)$.

$x=1, 2.5, 4.5, 6, 7.5$

2. Let $s = \int_0^t f(x)dx$ be the position of a particle moving along a coordinate axis [0,10]. The graph of f(t) is given.

$\therefore s' = f(x) = \text{velocity}$



a) What is the particle's velocity at t = 2? At t = 6?

3   0

b) Is the acceleration at t = 5.5 positive or negative? Justify your answer. since $f(x) = s' = \text{velocity}$ is increasing, acceleration is + @ 5.5

c) What is the particle's position at t = 0? t = 2? t = 5?

$s(0) = 0$ $s(2) = 6$ $s(5) = 5.5$

d) At what time does s have its largest value? Justify your answer. max

check $s(3) = 7.5$ ★ $s(10) = 2$

s has a max of 7.5 @ t = 3 b/c $s' = f$ goes from + to − @ t=3

e) When is the particle moving towards the origin? Away?

toward (3, 9)   away (0,3) ∪ (9, 10)

f) Find all t values of critical points of $s(t)$

when $s' = 0$ or is undefined

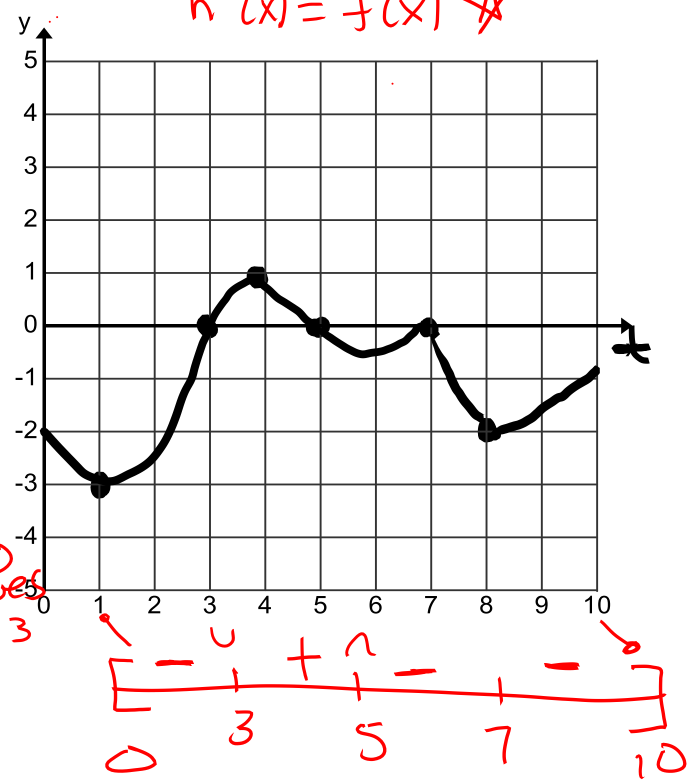
t = 3, 6, 9

g) Find all t values of inflection points of $s(t)$.

t = 5, 6, 8

3. Let h be the continuous function defined by $h(x) = \int_0^x f(t)dt$ [0,10]. The graph of f(x) is given below.

$h'(x) = f(x)$ ★



a) Find $f(1)$, $f(4)$ $f(1) = -3$ $f(4) = 1$

b) Is $h(2)$ positive or negative?

c) Is $h(5)$ positive or negative?

~~Justify your answer.~~

d) Find any x values where $h(x)$ has a relative max. Justify your answer. t=0, 5

e) Find any x values where $h(x)$ has a relative min. Justify your answer. h(x) has a rel. min @ t=3 b/c $h'(x) = f(x)$ goes from − to + @ t = 3

t=10 b/c t=10 is an endpt & $h'(x) = f(x) < 0$ to the left

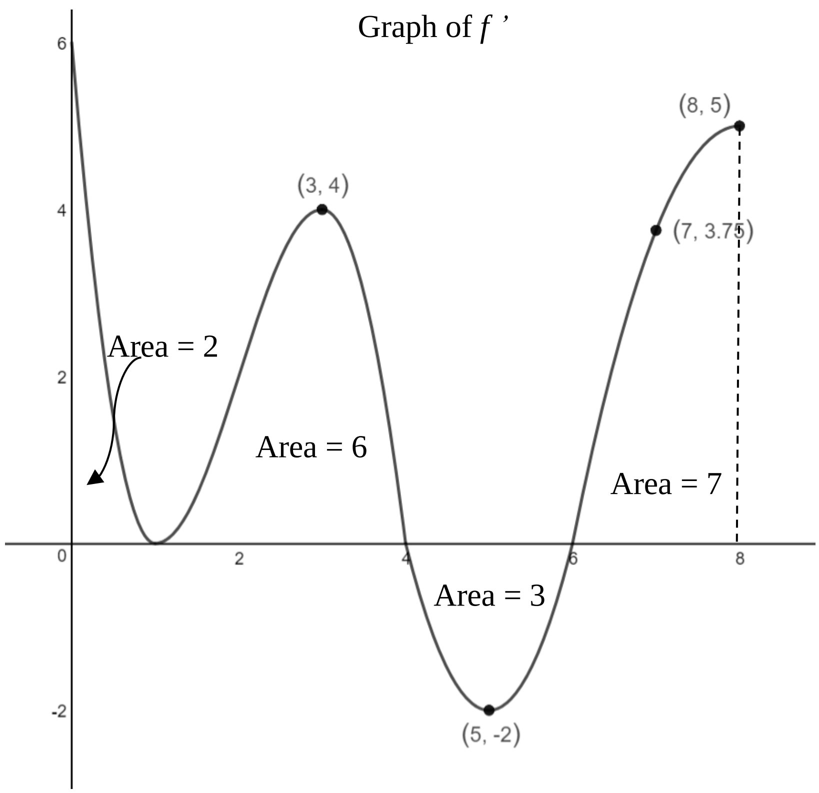
f) Find all x values of inflection points of $h(x)$

t = 1, 4, 6, 7, 8

b d. h(x) has a rel. max @ t=0 b/c t=0 is an endpt & $h'(x) = f(x) < 0$ to the right.

h(x) has a rel. max @ t=5 b/c $h'(x) = f(x)$ goes from + to − @ t=5

4.



The figure above shows the graph of f ’, the derivative of a twice- differentiable function f, on the closed interval $0 \le x \le 8$. The areas of the regions between the graph of f ’ and the x-axis are labeled in the figure.

* f(x) is the antiderivative of f' * you must understand this!

a. If $f(0) = 3$, write an expression for $f(x)$. Find $f(6)$

$\int_0^x f'(t)dt = f(x) - f(0)$

$f(x) = f(0) + \int_0^x f'(t)dt$

$f(6) = 3 + \int_0^6 f'(t)dt$

$= 3 + [2 + 6 - 3]$

$= 8$

b. If $f(4) = -7$, write an expression for $f(x)$. Then, find $f(1)$.

$\int_4^x f'(t)dx = f(x) - f(4)$

$f(x) = f(4) + \int_4^x f'(t)dx$

$f(1) = -7 + \int_4^1 f'(t)dt$

$= -7 - \int_1^4 f'(t)dt$

$f(1) = -7 - 6 = -13$

c. If $f(8) = -2$, write an expression for $f(x)$. Then, find $f(0)$

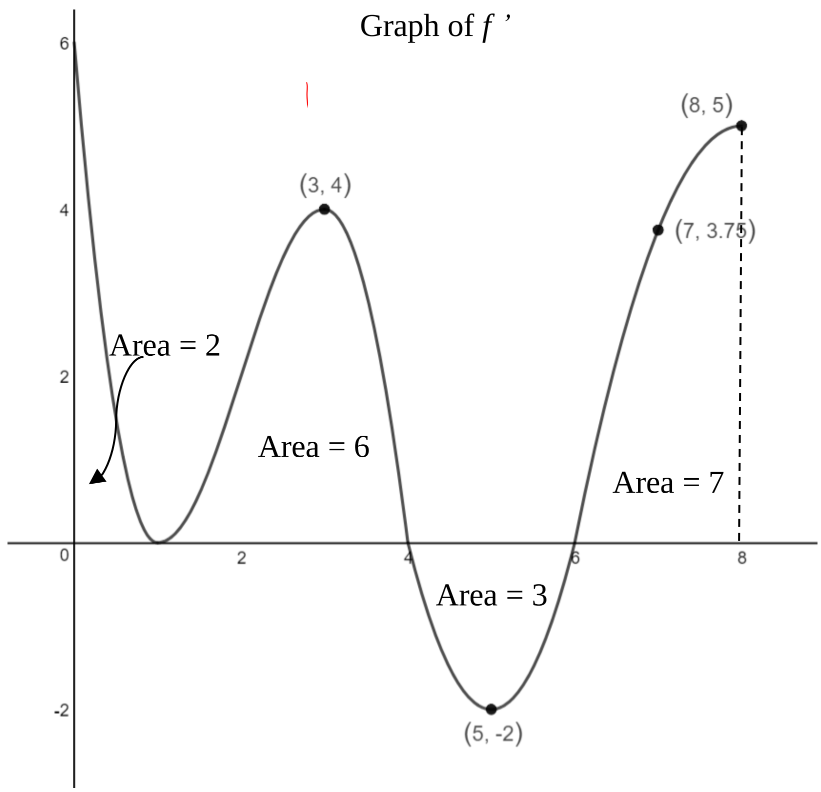
$\int_8^x f'(t)dt = f(x) - f(8)$

$f(8) + \int_8^x f'(t)dt = f(x)$

$f(0) = -2 + \int_8^0 f'(t)dt$

$= -2 - \int_0^8 f'(t)dt$

$f(0) = -2 - (12) = -14$



The figure above shows the graph of *f* ', the derivative of a twice- differentiable function *f*, on the closed interval $0 \le x \le 8$. The areas of the regions between the graph of *f* ' and the *x*-axis are labeled in the figure. Let $f(0) = 9$.

a. Write an expression for $f(x)$.

$$f(x) = f(0) + \int_0^x f'(t)\,dt$$

$$\int_0^x f'(t)\,dt = f(x) - f(0)$$

$$f(x) = 9 + \int_0^x f'(t)\,dt$$

b. Find the maximum value of $f(x)$ on $[0,8]$. Justify your answer.

f': [0 + 1 + 4 − 6 + 8]

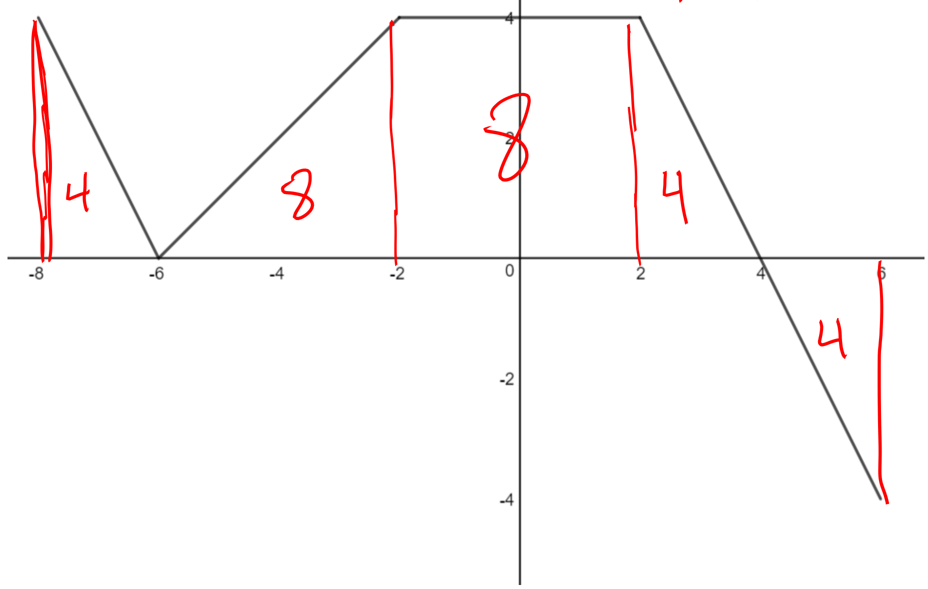
Check all

x	$f(x)$
0	$9+0=9$
4	$9+8=17$
6	$9+5=14$
☆ 8	$9+12=21$

$f(x)$ has a max of 21 @ $x=8$ b/c $x=8$ is an endpt and $f' > 0$ to the left of $x=8$.

Let p be the antiderivative of f and $p(-2) = -8$.

Graph of f $= p'(x)$



a. Write an expression for $p(x)$.

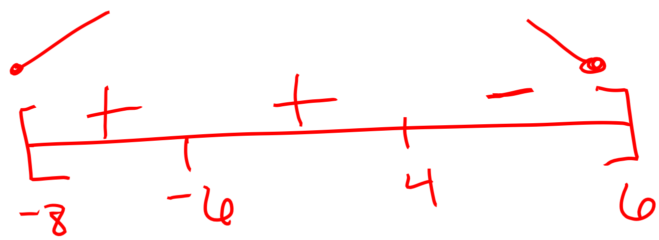
$$\int_{-2}^{x} f(t)\,dt = p(x) - p(-2)$$

$$p(-2) + \int_{-2}^{x} f(t)\,dt = p(x)$$

$$-8 + \int_{-2}^{x} f(t)\,dt = p(x)$$

b. Find the minimum value of $p(x)$ on [-8, 6]. Justify your answer.

$f = p'(x)$



Check all

x	$p(x)$
-8	-20 *
4	4
-6	-16
6	0

$$p(-8) = -8 + \int_{-2}^{-8} f(t)\,dt$$

$$= -8 - \int_{-8}^{-2} f(t)\,dt = -8 - 12$$

$$p(4) = -8 + \int_{-2}^{4} f(t)\,dt$$

$$= -8 + 12$$

$$p(-6) = -8 - \int_{-6}^{-2} f(t)\,dt$$

$$= -8 - 8$$

$$p(6) = -8 + \int_{-2}^{6} f(t)\,dt$$

$$-8 + 12 - 4 = 0$$

$p(x)$ has a min of -20 @ $x = -8$ b/c $x = -8$ is an endpt and $p'(x) = f(x) > 0$ to the right of $x = -8$.