

Section 6.4 Notes

Fundamental Theorem Part 1 Graph Problems

Let $F(x) = \int_0^x f(t) dt$ where f is the function graphed below (all segments and a semicircle).

$F'(x) = \frac{d}{dx} \int_0^x f(t) dt = f(x) \cdot 1$

a) Evaluate $F(-2)$, $F(0)$, $F(2)$, and $F(7)$

$F(-2) = \int_0^{-2} f(t) dt = - \int_{-2}^0 f(t) dt = -(-\pi) = \pi$

$F(2) = \int_0^2 f(t) dt = \frac{1}{2}(2)(2) = 2$

$F(0) = \int_0^0 f(t) dt = 0$

$F(7) = \int_0^7 f(t) dt = \frac{1}{2}(4)(2) - \frac{1}{2}(3)(3) = \frac{8}{2} - \frac{9}{2} = \frac{-1}{2}$

b) Identify all relative extrema of F in the interval $(-6, 7)$. Justify your answers.

$\star F'(x) = f(x) = 0$ $F(x)$ has a rel. max @ $x=4$ since $F'(x)=f(x)$ changes from + to - @ $x=4$ & $x=-4$.

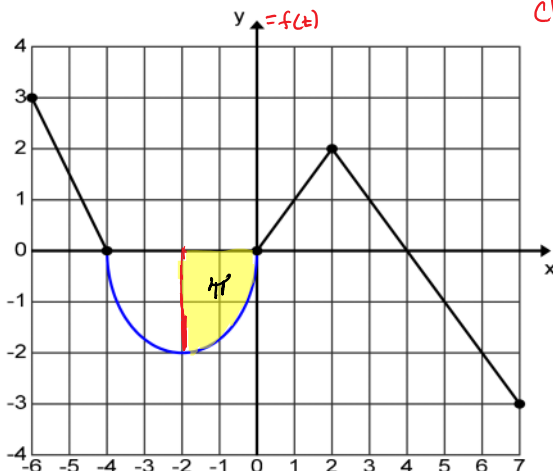
c.p. $x=-4, 0, 4$

$F(x)$ has a rel. min @ $x=0$ since $F'(x)=f(x)$ changes from - to + @ $x=0$.

c) Identify all x-coordinates of the inflection points of F in the interval $[-6, 7]$. Justify your answer.

$F''(x) = f'(x) \Rightarrow$ sign change

F has p.o.i.s @ $x = \pm 2$ since $F'' = f'$ changes signs @ $x = \pm 2$.



Let $F(x) = \int_0^x f(t) dt$ where f is the function graphed below (all segments and a semicircle).

$[-8, 6]$

a) Evaluate $F(-8)$, $F(-3)$, $F(0)$, $F(5)$, and $F(6)$

$$F(-8) = \int_0^{-8} f(t) dt = - \int_{-8}^0 f(t) dt = -\frac{9}{4}\pi - 12$$

$$F(5) = \int_0^5 f(t) dt = \frac{9}{4}\pi - 3$$

b) Identify all relative extrema of F on $[-8, 6]$. Justify your answer.

$$F'(x) = f(x) = 0$$

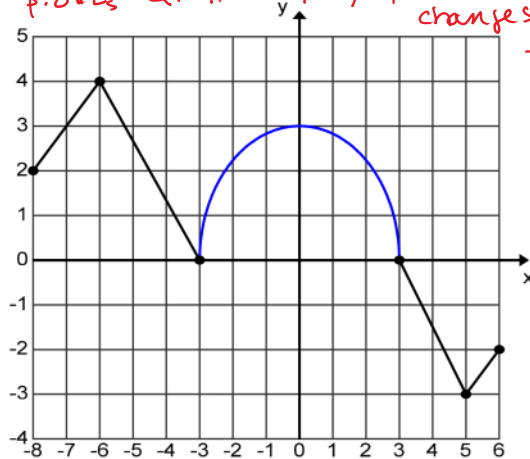
$$\text{c.p. } x = \pm 3$$

F has a rel max @ $x = 3$
since $F'(x) = f(x)$ changed from
+ to - @ $x = 3$

c) Identify all x-coordinates of the inflection points of F in the interval $[-8, 6]$. Justify your answer.

$$F''(x) = f'(x) \Rightarrow \text{changes s/s h}$$

F has p.o.i.s at $x = -6, -3, 0, 5$ since $F'' = f'$
changes signs at those pts.



F has a rel min @ $x = -8$
b/c $x = -8$ is an endpt
and $F' = f > 0$ to the right
of $x = -8$.

F has a rel min @ $x = 6$
b/c $x = 6$ is an endpt.
and $F' = f < 0$ to the left
of $x = 6$.