6.4 Day 2

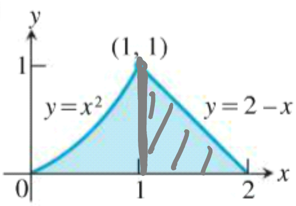
Wednesday, January 24, 2018 7:36 AM

FTC part 1

Homework questions

In Exercises 45–48, find the area of the shaded region.

45.

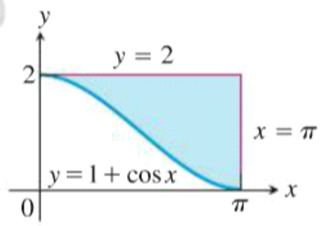


$$\int_0^1 x^2\,dx + \underbrace{\int_1^2 (2-x)\,dx}$$

$$\frac{x^3}{3}\Big|_0^1 + \frac{1}{2}$$

$$\frac{1}{3} - 0 + \frac{1}{2} = \boxed{5/6}$$

47.



Part

$$2\pi - \underbrace{\int_0^{\pi} (1+\cos x)\,dx}_{\text{blue Region}}$$

$$\underbrace{\int_0^{\pi} (1+\cos x)\,dx}_{\text{white region}}$$

Recall FTC part II

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f(x)

ex: $\displaystyle\int_{-1}^{2} (3x^2 - 1)\,dx = x^3 - x \Big|_{-1}^{2} = 8 - 2 - 0 = \boxed{6}$

Let's Explore . . .

$f(t)=t^2$

Let $A(x)=\int_1^x t^2\,dt$

1. $A(1)=\int_1^1 t^2\,dt = 0$

2. $A(3)=\int_1^3 t^2\,dt = \frac{t^3}{3}\Big|_1^3 = 9-\frac{1}{3} = 8\frac{2}{3} = \frac{26}{3}$

3. $A(x)=\int_1^x t^2\,dt = \frac{t^3}{3}\Big|_1^x = \underset{F(x)}{\frac{x^3}{3}} - \underset{F(1)}{\frac{1}{3}}$

$F(x) - F(1)$

★ 4. hmmm..... $A'(x) = \frac{d}{dx}\underbrace{\int_1^x t^2\,dt} = \frac{d}{dx}\left[F(x)-F(1)\right]$

$= \frac{d}{dx}F(x) - \frac{d}{dx}F(1)$

$= f(x) - 0$

$\boxed{A'(x) = x^2}$

Fundamental Theorem of Calculus Part 1

If $f(x)$ is continuous on $[a,b]$

and we define $F(x)=\int_a^x f(t)\,dt$, then...

$\leftarrow$ a function

$F'(x) = \frac{d}{dx}\int^x f(t)\,dt = f(x)$

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

↑ any constant

why? $\frac{d}{dx}\int_a^x f(t)\,dt = \frac{d}{dx}\left[F(x) - F(a)\right]$

$f(x) - 0$

$$\boxed{f(x)}$$

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examples

a. $y = \int_2^x (t^3 - 8)\,dt$ find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{t^4}{4} - 8t\right]_2^x = \frac{d}{dx}\left[\frac{x^4}{4} - 8x - (-12)\right]$$

$$\boxed{\frac{dy}{dx} = x^3 - 8}$$

b. $y = \int_{119}^x 3t\sin t\,dt$ $\boxed{\frac{dy}{dx} = 3x\sin x}$

c. $y = 2x^4 + \int_x^{\pi} \ln t^2\,dt$

$$\frac{dy}{dx} = 8x^3 + \frac{d}{dx}\left[F(\pi) - F(x)\right]$$

$$= 8x^3 + 0 - f(x)$$

$$= 8x^3 - \ln x^2$$

$$y = 2x^4 - \int_{\pi}^{x} \ln t^2$$

$$\frac{dy}{dx} = 8x^3 - \ln x^2$$

What happens now?! find y'

$$y = \int_{3}^{x^2} \cos t^3 \, dt$$

$$y' = \frac{d}{dx}\int_{3}^{x^2} \cos t^3 \, dt = \frac{d}{dx}\left[F(x^2) - F(3)\right]$$

$$f(x^2) \cdot 2x - 0$$

$$y' = \cos(x^2)^3 \cdot 2x$$

$$\boxed{= \cos x^6 \cdot 2x}$$

extension $A(x) = \displaystyle\int_{a}^{g(x)} f(t)\,dt$ $\quad A'(x) = f(g(x)) \cdot g'(x)$

What Now?!!

x^2

What now . . .

$$y = \int_x^{x^2} \sin t \, dt \qquad \text{find } y'$$

$$y' = \frac{d}{dx} \int_x^{x^2} \sin t \, dt$$

$$= \frac{d}{dx}\left[F(x^2) - F(x)\right]$$

$$f(x^2) \cdot 2x - f(x)$$

$$\sin x^2 \cdot 2x - \sin x$$