1. Consider a particle with velocity given by the function $v(t) = 5 + 4t$. Find the position function $s(t)$ and the displacement (change in position) from $t = 0 \text{ to } t = 10$.

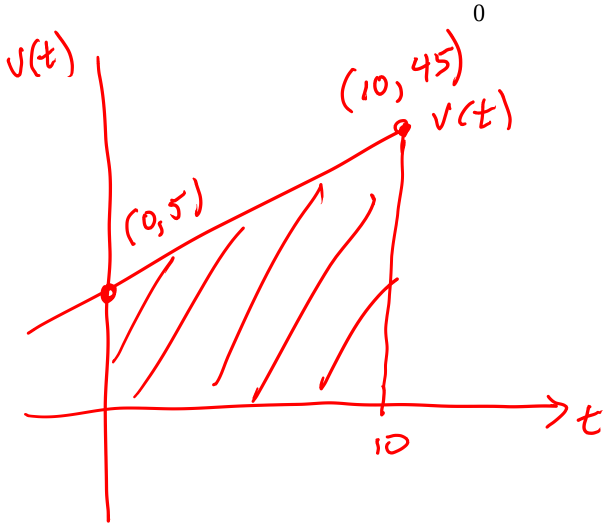
$s(t) = 5t + 2t^2 + C$

$s(10) = 5 \cdot 10 + 2 \cdot 10^2 + C = 250 + C$

$s(0) = 5 \cdot 0 + 2 \cdot 0 + C = C$

$\Delta s = s(10) - s(0) = 250 + C - C = 250$

2. Now, graph $v(t) = 5 + 4t$ and find $\int_0^{10} v(t)\,dt$.



$\int_0^{10} v(t)\,dt = \frac{1}{2} \cdot 10 \cdot (5 + 45)$

$= 250$

3. How is the displacement found in part 1 related to $\int_0^{10} v(t)\,dt$ found in part 2?

They are equal.

4. How is the position function related to the velocity function?

The position function is the antiderivative of the velocity.

CONCLUSION:

$$\text{For any function } f(t), \qquad \int_a^x f(t)\,dt = F(x) - F(a), \qquad \text{where } F(x) = \text{the antiderivative of } f(x).$$

Examples: Evaluate the integrals below using antiderivatives and the information above. Check your answers using **fnint** on your calculator.

1. $\int_{\pi}^{2\pi} \sin t\,dt$ $= -\cos(2\pi) - (-\cos(\pi)) = -1 - 1 = -2$

2. $\int_1^e \frac{1}{u}\,du$ $= \ln e - \ln 1 = 1 - 0 = 1$

3. $\int_1^4 -x^{-2}\,dx$ $= 4^{-1} - 1^{-1} = \frac{1}{4} - 1 = \frac{-3}{4}$

4. $\int_{-1}^{1} \frac{1}{1+x^2}\,dx$ $= \tan^{-1}(1) - \tan^{-1}(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

5. $\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx$ $= \sin^{-1}(1/2) - \sin^{-1}(0) = \frac{\pi}{6} - 0 = \frac{\pi}{6}$

6. $\int_1^e \frac{x^2+1}{x}\,dx$ $= \int_1^e \left(\frac{x^2}{x} + \frac{1}{x}\right)dx = \int_1^e \left(x + \frac{1}{x}\right)dx = \frac{1}{2}x^2 + \ln x \Big|_1^e$

$= \frac{1}{2}e^2 + \ln e - \left(\frac{1}{2}\cdot 1^2 + \ln 1\right)$

$= \frac{1}{2}e^2 + 1 - \frac{1}{2} = \frac{1}{2}e^2 - \frac{1}{2} = \frac{1}{2}(e^2 - 1)$