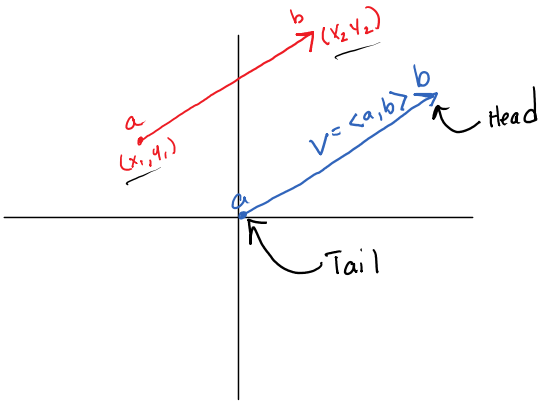
6.1 Day 1

Wednesday, April 17, 2019 8:55 AM



$\vec{v} = \langle a, b \rangle$

This is called the position vector of (a, b) and is denoted by $\langle a, b \rangle$

Two dimensional vector is an ordered pair denoted in component form as $\langle a, b \rangle$

☆ a & b are the components of vector $\vec{v}$

magnitude: of $\vec{v}$, denoted $|v|$, is the length of the arrow

* if v is represented by the arrow from (x_1, y_1) to (x_2, y_2) then $|v| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

if $v = \langle a, b \rangle = |v| = \sqrt{a^2 + b^2}$

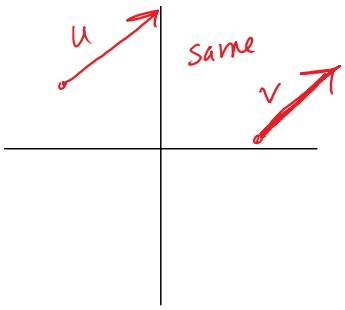
** the direction of v is the direction in which the arrow is pointing.

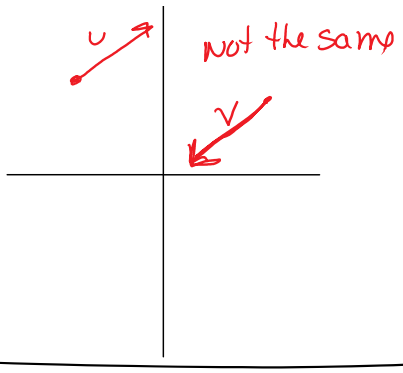
Zero vector $= \langle 0, 0 \rangle$ has zero length & no direction.

Important: any two arrows (vectors) with the same length & pointing in the same direction represent the same vector

$|u| = |v|$

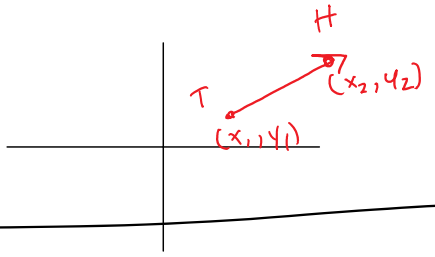
not the same





Head - minus - Tail (HMT) Rule

if an arrow has an initial point (x_1, y_1) and terminal point (x_2, y_2), it represents vector $\langle x_2 - x_1, y_2 - y_1 \rangle$



Example

R = (-4, 2) S (-1, 6) P (2, -1) Q (5, 3)

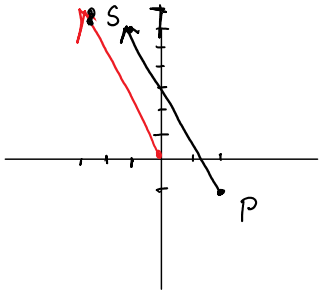
a. $\overrightarrow{RS}$

$\langle 3, 4 \rangle$

b. $\overrightarrow{PQ}$

$\langle 3, 4 \rangle$

c. Draw $\overrightarrow{PS} = u$



component form

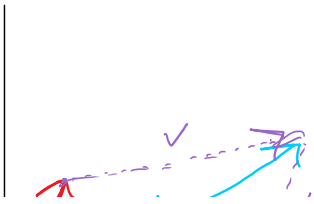
$\overrightarrow{PS}$ $\langle -3, 7 \rangle$

Magn. of $\overrightarrow{RS}$ = $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$

Adding vectors

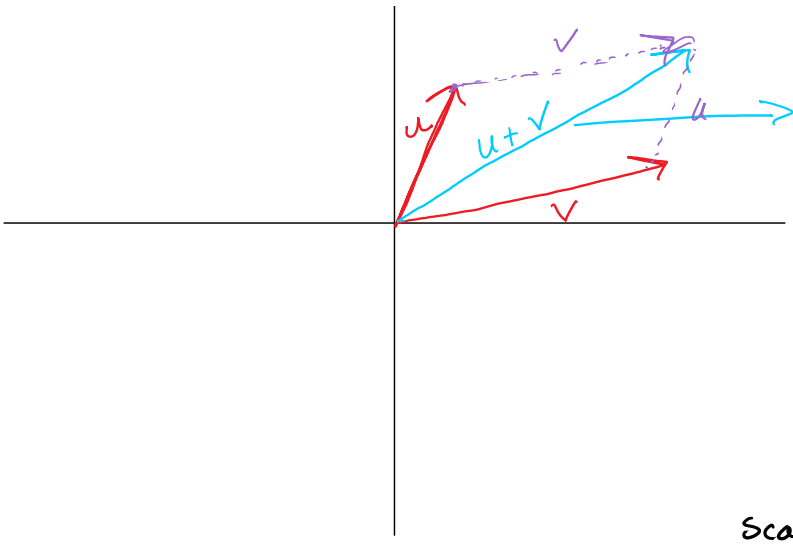
$u = \langle u_1, u_2 \rangle$ $v = \langle v_1, v_2 \rangle$

$u + v = \langle u_1 + v_1, u_2 + v_2 \rangle$



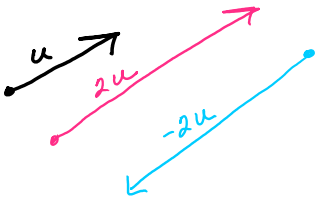
Unit vector

$u + v = \langle u_1 + v_1, u_2 + v_2 \rangle$



scalar multiplication

$u = \langle u_1, u_2 \rangle \quad 2u = \langle 2u_1, 2u_2 \rangle$



Examples

Let $u = \langle -1, 3 \rangle$ $v = \langle 4, 7 \rangle$

a. $u + v$

$\langle 3, 10 \rangle$

b. $3u$

$\langle -3, 9 \rangle$

c. $2u + -1v$

$\langle -6, -1 \rangle$

Unit Vectors:

A vector u with length $|u| = 1$ is a unit vector.

$V \neq$ zero vector, then the vector

$$u = \frac{v}{|v|} = \frac{1}{|v|} \cdot v \quad \text{unit vector}$$

example: Find the unit vector in the direction of $v = \langle -3, 2 \rangle$ and verify it has a length of 1.

$|v| = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$ $\quad \frac{v}{|v|} = \frac{1}{\sqrt{13}} \cdot \langle -3, 2 \rangle$

$\left\langle \frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$ verify: $\sqrt{\left(\frac{-3}{\sqrt{13}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2}$

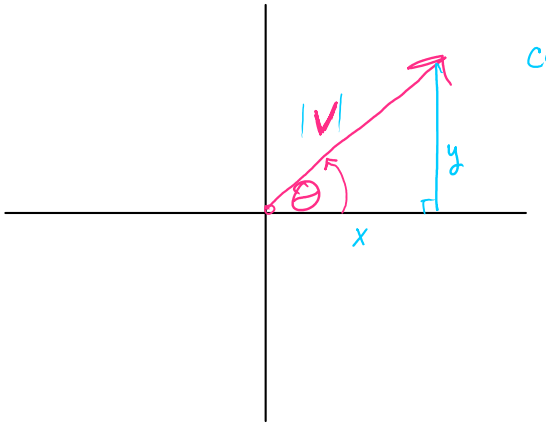
$\sqrt{\frac{9}{13} + \frac{4}{13}} = 1$ ☺

The two unit vectors $i = \langle 1, 0 \rangle$ $\quad j = \langle 0, 1 \rangle$ are standard unit vectors.

ex: $v = \left\langle \frac{-3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle \Rightarrow \frac{-3}{\sqrt{13}} i + \frac{2}{\sqrt{13}} j$

Direction Angles:

- made from the positive x-axis moving counter clockwise

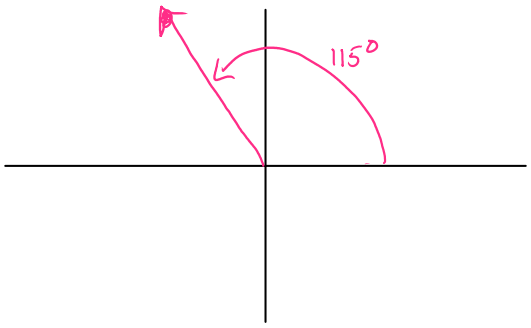


$\cos\theta = \frac{x}{|v|}$ $\quad x = |v| \cos\theta$

$\sin\theta = \frac{y}{|v|}$ $\quad y = |v| \sin\theta$

Ex: Find the components of vector v with the direction angle 115° and magnitude 6.

the direction angle 115 and magnitude 6



$\langle 6\cos 115^\circ, 6\sin 115^\circ \rangle$

$\langle -2.54, 5.44 \rangle$