Precalculus *Name:*
5.1 and 5.2 *period:*

To receive credit for the following problems, you must show each step in the process of arriving at your answer or verifying the identity.

1. **Simplify:** **a)** $\tan^2\theta\cos^2\theta + \cot^2\theta\sin^2\theta$ **b)** $\cos^2\theta(\sec^2\theta - 1)$

$\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\cancel{\cos^2\theta} + \frac{\cos^2\theta}{\cancel{\sin^2\theta}}\cdot\cancel{\sin^2\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta\cdot\sec^2\theta - \cos^2\theta$

$1 - \cos^2\theta$

$\sin^2\theta$

2. **Prove the following identities.**

a. $\csc\theta - \cot\theta = \dfrac{\sin\theta}{1+\cos\theta}$

$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$\frac{1-\cos\theta}{\sin\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}$

$\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}$

$\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta}$

b. $\dfrac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \dfrac{\tan\theta}{1-\tan^2\theta}$

$= \frac{\frac{\sin\theta}{\cancel{\cos\theta}}}{1 - \frac{\sin^2\theta}{\cancel{\cos^2\theta}}}\cdot\frac{\cos^2\theta}{\cos^2\theta}$

$\frac{\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

c. $\dfrac{\sec\theta}{1+\sec\theta} = \dfrac{1-\cos\theta}{\sin^2\theta}$

$= \frac{1-\cos\theta}{1-\cos^2\theta}$

$= \frac{1}{1+\cos\theta}$

$= \frac{1}{1+\frac{1}{\sec\theta}}\cdot\frac{\sec\theta}{\sec\theta}$

$= \frac{\sec\theta}{\sec\theta + 1} = \frac{\sec\theta}{1+\sec\theta}$

d. $\dfrac{\sec^2\theta - \tan^2\theta + \tan\theta}{\sec\theta} = \sin\theta + \cos\theta$

$\frac{1+\tan\theta}{\sec\theta}$

$\cos\theta\left(1 + \frac{\sin\theta}{\cos\theta}\right)$

$\cos\theta + \sin\theta$

$\sin\theta + \cos\theta$

e. $\dfrac{\sin\theta + \cos\theta}{\sin\theta} - \dfrac{\cos\theta - \sin\theta}{\cos\theta} = \sec\theta\csc\theta$

$\frac{\cos\theta}{\cos\theta}\cdot\frac{\sin\theta + \cos\theta}{\sin\theta} - \frac{\cos\theta - \sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta}$

$\frac{\sin\theta\cos\theta + \cos^2\theta - \sin\theta\cos\theta + \sin^2\theta}{\sin\theta\cos\theta}$

$\frac{1}{\sin\theta\cos\theta}$

$\sec\theta\csc\theta$

f. $\dfrac{1 - 2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta - \cot\theta$

$\frac{1 - \cos^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$

$\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$

$\frac{\sin^2\theta}{\sin\theta\cos\theta} - \frac{\cos^2\theta}{\sin\theta\cos\theta}$

$\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta} = \tan\theta - \cot\theta$

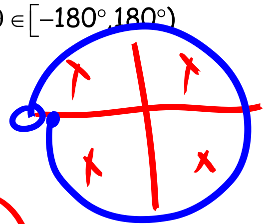
3. Solve each of the following equations in the indicated domain.

a) $3\tan^2\theta - 1 = 0$ $\theta \in [-180°, 180°)$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$



$\theta = \pm 30° + 180° n$

$\{30°, 150°, -30°, -150°\}$

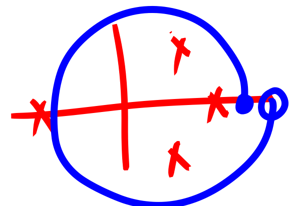
b) $2\sin x\cos x = \sin x$ $x \in [0, 2\pi)$

$2\sin x\cos x - \sin x = 0$

$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0$ or $2\cos x - 1 = 0$

$x = \sin^{-1}(0)$ $\cos x = \frac{1}{2}$ $x = \cos^{-1}\left(\frac{1}{2}\right)$



$x = 0 + \pi n$ or

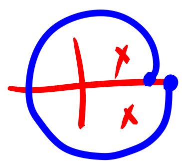
$x = \pm\frac{\pi}{3} + 2\pi n$

$\{0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\}$

c) $\sqrt{3}\sec 3\varphi - 2 = 0$ $\varphi \in [0°, 360°]$

$\sec 3\varphi = \frac{2}{\sqrt{3}}$

$\cos 3\varphi = \frac{\sqrt{3}}{2}$



$3\varphi = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pm 30° + 360° n$

$\varphi = \pm 10° + 120° n$

$\{10°, 130°, 250°, 110°, 230°, 350°\}$

d) $\cos^2 x - \sin^2 x + \cos x = 0$ $x \in [-2\pi, 2\pi]$

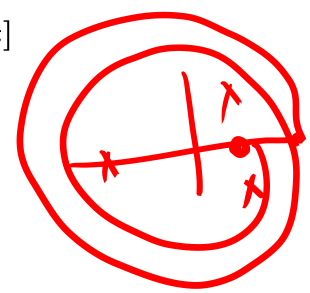
$\cos^2 x - (1 - \cos^2 x) + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1)$

$2\cos x - 1 = 0$ or $\cos x + 1 = 0$

$\cos x = \frac{1}{2}$ or $\cos x = -1$



$x = \pi + 2\pi n$ or

$\pm\frac{\pi}{3} + 2\pi n$

$\{\pi, -\pi, \frac{\pi}{3}, -\frac{\pi}{3}, \frac{5\pi}{3}, -\frac{5\pi}{3}\}$

e) $\cos\theta + \sin\theta + 1 = 0$ $\theta \in [-360°, 360°]$

$\cos\theta + 1 = -\sin\theta$

$\cos^2\theta + 2\cos\theta + 1 = \sin^2\theta$

$= 1 - \cos^2\theta$

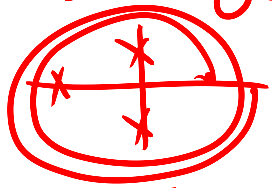
$2\cos^2\theta + 2\cos\theta = 0$

$2\cos\theta(\cos\theta + 1) = 0$

$2\cos\theta = 0$ or $\cos\theta = -1$

$\cos\theta = 0$ $\theta = \cos^{-1}(-1)$

$\theta = \cos^{-1}(0)$



$\theta = 180° + 360° n$

or

$\theta = 90° + 180° n$

$\{180°, -180°, 90°, 270°, -90°, -270°\}$