9.8 Day 2

Friday, February 28, 2020 10:34 AM

pg 654: 11, 13, 15, 17, 26, 55b, 56, 73

11. $\sum_{n=0}^{\infty} \left(\frac{x}{4}\right)^n$ Geometric $\left|\frac{x}{4}\right| < 1$ converges

so $-1 < \frac{x}{4} < 1$ converges abs $(-4, 4)$

13. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right| = \lim_{n\to\infty} \frac{x \cdot n}{n+1} = \lim_{n\to\infty} x = x \quad -1 < x < 1$

check endpts! $x = 1$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ = alt. series

$\frac{1}{n} \geq 0$ ✓

$\frac{1}{n} \geq \frac{1}{n+1}$ ✓

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

when $x = -1$ $\sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{n}$

$= \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test

so... converges on $(-1, 1]$

converges abs. on $(-1, 1)$

conv. cond. @ $x = 1$

15. $\sum_{n=0}^{\infty} \frac{x^{5n}}{n!}$ $\lim_{n\to\infty} \left(\frac{x^{5n+5}}{(n+1)!} \cdot \frac{n!}{x^{5n}} \right) = \lim_{n\to\infty} \frac{x^5}{n+1} = 0$ $R = \infty$

$\therefore$ converges $(-\infty, \infty)$

converges abs. $(-\infty, \infty)$

17. $\sum_{n=0}^{\infty} (2n)! \left(\frac{x}{3}\right)^n$ $\lim_{n\to\infty} \frac{(2n+2)! \left(\frac{x}{3}\right)^{n+1}}{(2n)! \left(\frac{x}{3}\right)^n} = \lim_{n\to\infty} (2n+2)(2n+1) \left(\frac{x}{3}\right) = \infty$

$\therefore$ the series only converges @ $x = 0$

26. $\sum^{\infty} \frac{(-1)^n x^{2n+1}}{\ldots}$ $\lim \left| (-1)^{n+1} \cdot x^{2n+3} \cdot \frac{2n+1}{\ldots} \right| = \lim \left| \frac{(-1)^1 x^3 \cdot (2n+1)}{\ldots} \right|$

26. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ $\quad \displaystyle\lim_{n\to\infty} \left| \frac{(-1)^{n+1} \cdot x^{2n+3}}{2n+3} \cdot \frac{2n+1}{(-1)^n \cdot x^{2n+1}} \right| = \lim_{n\to\infty} \left| \frac{(-1)^1 x^3 \cdot (2n+1)}{(2n+3)\cdot x} \right|$

$= \displaystyle\lim_{n\to\infty} \frac{x^3 \cdot (2n+1)}{(2n+3)\cdot x} = \lim_{n\to\infty} x^2 = x^2 \quad -1 < |x^2| < 1$

check $x = -1$ $\quad \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (-1)^{2n+1}}{2n+1}$ $\quad \displaystyle\lim_{n\to\infty} \frac{1}{2n+1} = 0$ ✓ $\quad (-1)^{3n+1}$

alternating series $\quad \frac{1}{2n+1} \geq \frac{1}{2n+3}$ ✓ $\quad (-1)^n \cdot 1$

$\frac{1}{2n+1} \geq 0$ ✓

$(-1)^n \cdot (-1)^{2n+1}$

check $x = +1$ $\quad \displaystyle\sum \frac{(-1)^n}{2n+1}$ $\quad$ alt. series converges

Converges on $[-1, 1]$
converges abs $(-1, 1)$
conv. cond $x = -1, 1$

55 b $\quad$ one possibility $\quad \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}$

56. a) iii $\quad$ b) i $\quad$ c) ii $\quad$ d) iv

* remember this is a sum of terms.

73. False

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n 2^n}$ $\quad$ converges for $x = 2$, but diverges for $x = -2$.