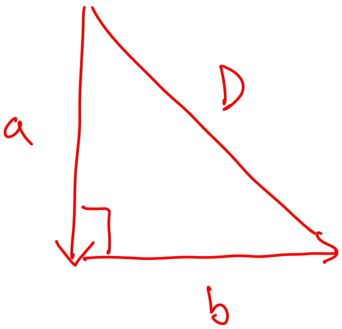
5.6b Related Rates examples

A Highway Chase

A police cruiser, approaching an intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph when they use the radar, then how fast is the speeder going?



$$D^2 = a^2 + b^2 \qquad D = \sqrt{a^2+b^2}$$

$$a = .6 \qquad b = .8 \quad \Rightarrow \quad D = \sqrt{.6^2+.8^2} = 1$$

$$\frac{da}{dt} = -60 \qquad \frac{dD}{dt} = 20$$

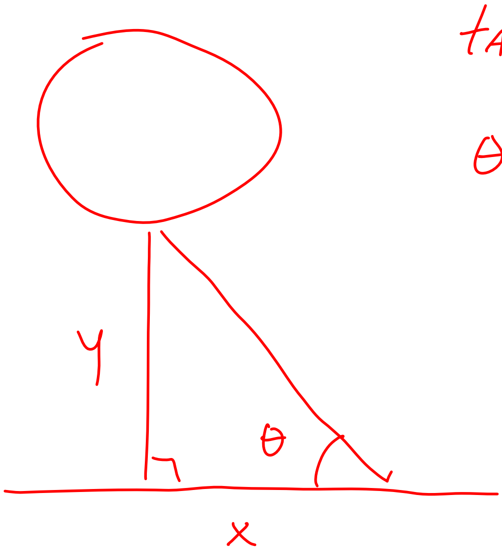
$$2D\frac{dD}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$$

$$2(1)(20) = 2(.6)(-60) + 2(.8)\frac{db}{dt}$$

$$\frac{db}{dt} = \frac{112}{1.6} = 70 \text{ mph}$$

A Rising Balloon

A hot-air balloon rising up from a level field is tracked by a range finder 500 from the liftoff point. At the moment the range finder's angle of elevation is $\frac{\pi}{4}$, that angle is increasing at a rate of 0.14 rad/min. How fast is the balloon rising at that moment?



$$\tan\theta = \frac{y}{x} \quad \Rightarrow \quad y = x\tan\theta$$

$$\theta = \frac{\pi}{4} \qquad \frac{d\theta}{dt} = 0.14 \qquad x = 500 \qquad y = 500\tan\frac{\pi}{4}$$

$$y = 500$$

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta + x\sec^2\theta\frac{d\theta}{dt}$$

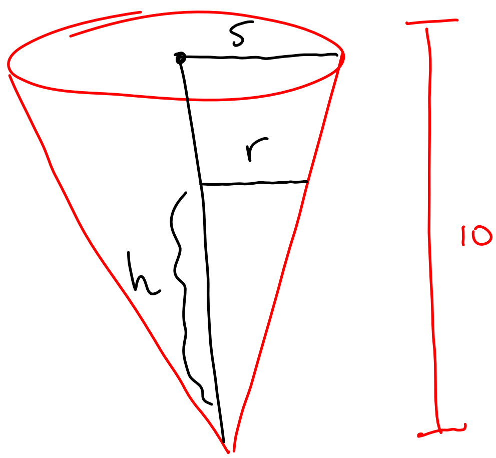
$$= 0\cdot\tan\frac{\pi}{4} + 500\sec^2\left(\frac{\pi}{4}\right)(.14)$$

$$= 500\left(\frac{2}{\sqrt{2}}\right)^2(.14)$$

$$= 140 \text{ ft/min}$$

Filling a Conical Tank

Water runs into a conical tank at the rate of $9\frac{ft^3}{\text{min}}$. The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep? How fast is the radius at the surface of the water increasing?



$$\frac{r}{h} = \frac{5}{10} \Rightarrow r = \frac{1}{2}h$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h = \frac{1}{12}\pi h^3$$

$$h = 6 \quad \frac{dV}{dt} = 9$$

$$\frac{dV}{dt} = \frac{1}{12}\cdot\pi\cdot 3h^2\cdot\frac{dh}{dt}$$

$$9 = \frac{1}{12}\cdot\pi\cdot 3\cdot 6^2\cdot\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{\pi}\ \text{ft/MIN}$$

Notice, that since $r = \frac{1}{2}h$

$$\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt} = \frac{1}{2\pi}\ \text{ft/MIN.}$$