Precalculus **Section 5.6 – The Law of Cosines**

The **Law of Sines** can be used to solve triangles when you know ASA, AAS.

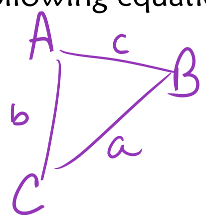
The **Law of Cosines** can be used to solve triangles when you know SAS, SSS.

(Either rule can be used for ASS, but remember that there could be 0, 1, or 2 triangles – we'll deal with that later.)

The Law of Cosines is called the "generalized Pythagorean Theorem."

The **Law of Cosines** states:

In any ΔABC with angles A, B, and C opposite sides a, b, and c, respectively, the following equations are true:



$$a^2 = b^2 + c^2 - 2bc\cos A$$

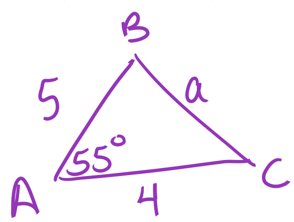
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Examples: Find the missing side.

1. $\Delta ABC, b = 4, c = 5, m\angle A = 55°$

SAS

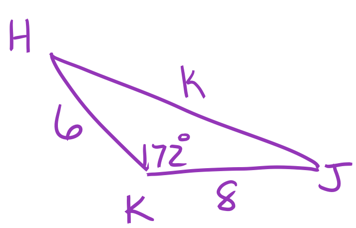


$$a^2 = 5^2 + 4^2 - 2(5)(4)\cos 55°$$

$$\boxed{a \approx 4.2}$$

2. $\Delta HJK, h = 8, j = 6, m\angle K = 172°$

SAS



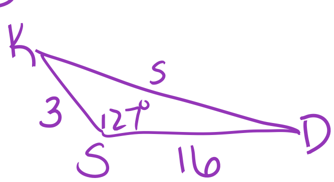
$$k^2 = 6^2 + 8^2 - 2(6)(8)\cos 172°$$

$$\boxed{k \approx 14.0}$$

Try it! Find the missing side.

3. $\Delta KSD, m\angle S = 127°, k = 16, d = 3$

SAS



$$s^2 = 3^2 + 16^2 - 2(3)(16)\cos 127°$$

$$\boxed{s \approx 18.0}$$

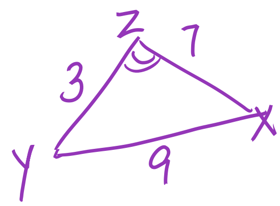
Find the angles of the triangle.

To Find $\angle Z$:

To Find $\angle X$:

4. $\Delta XYZ, x = 3, y = 7, z = 9$

SSS



$9^2 = 3^2 + 7^2 - 2(3)(7)\cos Z$

$23 = -42\cos Z$

$-\frac{23}{42} = \cos Z$

$\angle Z = \cos^{-1}\left(-\frac{23}{42}\right) \approx \boxed{123.2^\circ}$

$3^2 = 7^2 + 9^2 - 2(7)(9)\cos X$

$-121 = -126\cos X$

$\frac{121}{126} = \cos X$

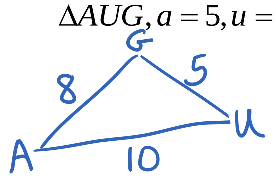
$\angle X = \cos^{-1}\left(\frac{121}{126}\right) \approx \boxed{16.2^\circ}$

Subtract from 180° to find $\angle Y$:

$\angle Y = 180 - \angle Z - \angle X \approx \boxed{40.6^\circ}$

Try it! Find the angles of the triangle.

5. $\Delta AUG, a = 5, u = 8, g = 10$



To Find $\angle G$:

$10^2 = 8^2 + 5^2 - 2(8)(5)\cos G$

$-\frac{11}{80} = \cos G$

$\boxed{\angle G = 97.9^\circ}$

To Find $\angle U$:

$8^2 = 5^2 + 10^2 - 2(5)(10)\cos U$

$\frac{-61}{-100} = \cos U$

$\boxed{\angle U = 52.4^\circ}$

To Find $\angle A$:

$\angle A = 180^\circ - \angle G - \angle U$

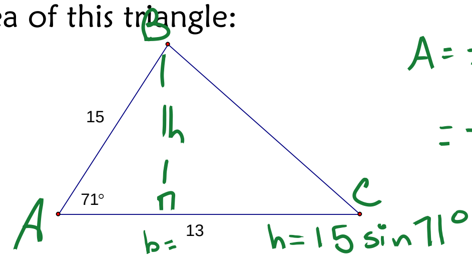
$\boxed{\angle A = 29.7^\circ}$

Assign p. 494
1, 3, 6, 9, 35, 38

Area of a Triangle – 2 Formulas

Area of a Triangle = $\frac{1}{2}bh$

Find the area of this triangle:



$A = \frac{1}{2}bh$

$= \frac{1}{2}(13)(15\sin 71^\circ)$

$= 92.2\ u^2$

(any side can be base)

Area of a Triangle

$$A_\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$$

This formula is for when you know 2 sides and included angle SAS

If you know 3 sides of the Δ (SSS) instead, you can use this formula from Geometry:

Heron's Formula

semi-perimeter

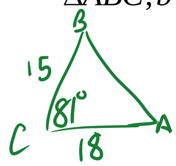
$s = \frac{a+b+c}{2}$

$A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

This formula is for when you know SSS

Examples: Find the area of the given triangle to the nearest 10^{th}.

6. $\Delta ABC, b = 18, a = 15, m\angle C = 81°$

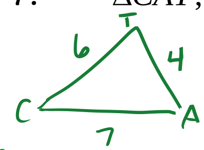


SAS

$A_\Delta = \frac{1}{2}(18)(15)\sin 81°$

$\approx \boxed{133.3\ u^2}$

7. $\Delta CAT, c = 4, a = 6, t = 7$



SSS

$s = \frac{6+4+7}{2} = 8.5$

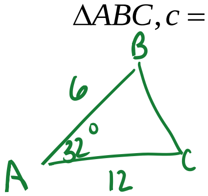
$A_\Delta = \sqrt{8.5(8.5-6)(8.5-4)(8.5-7)}$

$\approx \boxed{12.0\ u^2}$

Assign p. 494 #17, 20, 21, 24

Try it! Find the area of the given triangle to the nearest 10^{th}.

8. $\Delta ABC, c = 6, b = 12, m\angle A = 32°$

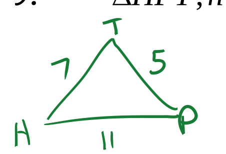


SAS

$A_\Delta = \frac{1}{2}(6)(12)\sin 32°$

$\approx \boxed{19.1\ u^2}$

9. $\Delta HPT, h = 5, p = 7, t = 11$

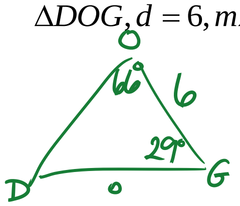


$s = \frac{7+5+11}{2} = 11.5$

$A_\Delta = \sqrt{11.5(11.5-7)(11.5-5)(11.5-11)}$

$\approx \boxed{13.0\ u^2}$

10. $\Delta DOG, d = 6, m\angle O = 66°, m\angle G = 29°$ (hint: how can you find the side you need first?)



ASA - Law of Sines to find o.

$m\angle D = 85°$

$\frac{\sin 85°}{6} = \frac{\sin 66°}{o}$

$o = 5.5$

$A_\Delta = \frac{1}{2}(5.5)(6)\sin 29°$

$\approx \boxed{8.0\ u^2}$