## 4.6 Day 3

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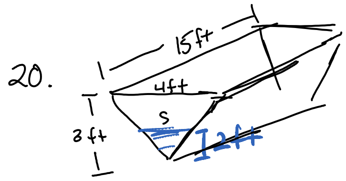
Wednesday, December 12, 2012
12:48 PM

20.




$\frac{dV}{dt} = 2.5\ ft^3/min \qquad h = 2ft \qquad \frac{dh}{dt}$

fixed (will never change)

$V = \frac{1}{2} b h \cdot (15)$

$\frac{b}{h} = \frac{4}{3}$ similar triangles

$b = \frac{4}{3}h$

$V = \frac{1}{2}\left(\frac{4}{3}h\right) \cdot h\,(15)$

$V = \frac{15}{2} \cdot \frac{4}{3} h^2$

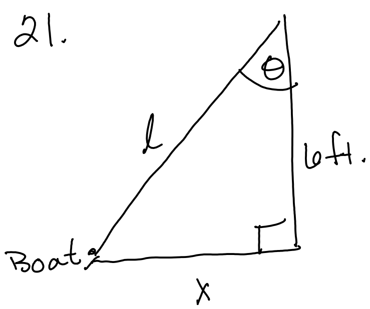
$\frac{dV}{dt} = 15 \cdot \frac{4}{3} h \frac{dh}{dt}$

$2.5 = \frac{60}{3}(2)\frac{dh}{dt}$

$\frac{dh}{dt} = .0625\ ft/min$ ☆

☆ The water level increases at the rate .0625 ft/min.

21.




l = length of rope

x = horizontal distance from the boat to the dock.

θ = angle between the rope and vertical line

$l^2 = x^2 + 36$

$100 = x^2 + 36$

$x = 8$

a. $\frac{dl}{dt} = -2\ ft/sec \qquad l = 10ft \qquad x = 8$

$\frac{dx}{dt} = ?$

$x^2 + 6^2 = l^2$

$2x\frac{dx}{dt} = 2l\frac{dl}{dt} \qquad x\frac{dx}{dt} = l\frac{dl}{dt}$

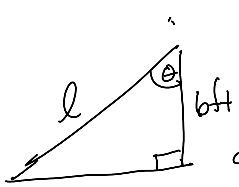
$8\frac{dx}{dt} = 10(-2)$

$\frac{dx}{dt} = \frac{-20}{8} = -\frac{5}{2}\ ft/sec$

$= -2.5\ ft/sec$ ☆

☆ The boat is approaching the dock at the rate of 2.5 ft/sec

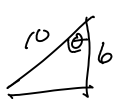
b. $\frac{d\theta}{dt} = ?$




$l = 10ft \qquad \frac{dl}{dt} = -2\ ft/sec$

$\cos\theta = \frac{6}{l} \qquad l\cos\theta = 6 \qquad -l\sin\theta\frac{d\theta}{dt} + \cos\theta\frac{dl}{dt} = 0$

$-10\sin(.927)\frac{d\theta}{dt} = -\cos(.927)(-2)$




$\cos\theta = \frac{6}{10}$

$\theta = \cos^{-1}(6/10) = 0.927$ store this

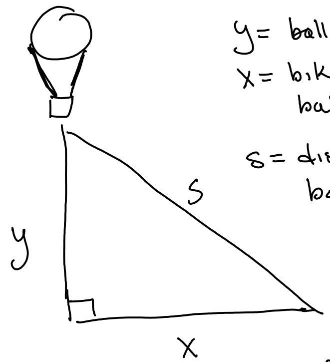
$\frac{d\theta}{dt} = \frac{(-\cos(.927)(-2))}{(-10\sin(.927))} = -.15\ rad/sec$

$\theta = \cos^{-1}(6/10) \approx \underbrace{0.927}_{\text{store this value!!}}$

$\frac{d\theta}{dt} = \frac{(\ldots)}{(-10\sin(.927))} = -.15 \text{ rad/sec}$

$\frac{d\theta}{dt} = \frac{-3}{20} = -.15 \text{ rad/sec}$ ☆

22.




y = balloon's height
x = bike's distance from balloon's starting point
s = distance between balloon & bike.

$\frac{dy}{dt} = 1$ ft/sec  $y = 65$ feet

$\frac{dx}{dt} = 17$ ft/sec  $\frac{ds}{dt} = ?$

3 secs before $x = 0$  $y = 65$

$y = 65 + 1(3) = 68$

$x = 0 + 17(3) = 51$

$68^2 + 51^2 = s^2$

$s = 85$

$x^2 + y^2 = s^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = s\frac{ds}{dt}$

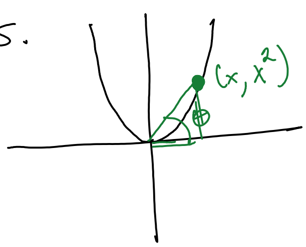
$51(17) + 68(1) = 85\frac{ds}{dt}$

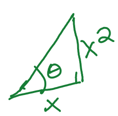
$\frac{935}{85} = \frac{ds}{dt}$

$\frac{ds}{dt} = 11$ ft/sec ☆

☆ The distance between the balloon and the bike is increasing at the rate of 11ft/sec.

25.




$\frac{dx}{dt} = 10$ m/sec  $\frac{d\theta}{dt} = ?$  $x = 3$

$\tan\theta = \frac{x^2}{x}$  $\tan\theta = x$  $\sec^2\theta\frac{d\theta}{dt} = \frac{dx}{dt}$

$\tan\theta = 3$

$\theta = \tan^{-1}(3)$

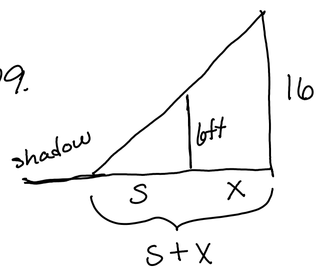
$\theta \approx \underbrace{1.249}_{\text{must store this value!!}}$

$(\sec 1.249)^2\frac{d\theta}{dt} = 10$

$\frac{d\theta}{dt} = \frac{10}{(\sec 1.249)^2} = 1$ rad/sec ☆

☆ The angle of inclination is increasing at the rate of 1 rad/sec.

29.




x = man's distance from streetlight $\qquad \frac{dx}{dt} = -5\text{ ft/sec} \qquad x = 10$

s = length of shadow $\qquad \frac{ds}{dt} = ?$

Simliar triangles $\quad \frac{s+x}{16} = \frac{s}{6}$

$$6(s+x) = 16s$$

$$6s + 6x = 16s$$

$$\frac{6x}{10} = \frac{10s}{10}$$

$$s = \frac{3}{5}x$$

$$\frac{ds}{dt} = \frac{3}{5}\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{3}{5}(-5) = -3\text{ ft/sec} ☆$$

☆ The shadow length is changing at the rate of -3ft/sec