5.6 Day 1

Friday, November 30, 2018 10:20 AM

5.6a Notes Related Rates

Assume x and y are both functions of time. That is, assume they both vary or change over time.

Differentiate:

a) $y = 2x^2 + 5x$ with respect to x

$\frac{dy}{dx} = 4x + 5$

a) $y = 2x^2 + 5x$ with respect to t

$\frac{dy}{dt} = 4x \cdot \frac{dx}{dt} + 5 \cdot \frac{dx}{dt}$

Given $y = xz$

a) Find $\frac{dy}{dt}$ if z is a constant and x is a function of time.

$y = x\boxed{z}$

$\frac{dy}{dt} = z\frac{dx}{dt}$

b) Find $\frac{dy}{dt}$ if x is a constant and z is a function of time.

$y = \boxed{x}z$ $\frac{dy}{dt} = x\frac{dz}{dt}$

c) Find $\frac{dy}{dt}$ if x and z are both functions of time.

$\frac{dy}{dt} = x\frac{dz}{dt} + z\frac{dx}{dt}$

Related rates problems require determining how one variable is changing with time as other related variables are changing with time.

Examples:

How the depth of water in a bathtub is changing as the volume is changing.

How the volume of a balloon is changing as the radius is changing

How the current in an electrical circuit is changing as the voltage is changing

Important Note: Pay attention to the sign of rates of change in related rates problems.

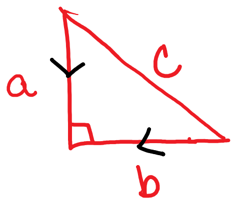
Example:

$\frac{dV}{dt} =$ the rate of change of the volume of water in a tub with respect to time.

a) If the water is flowing into the tub we would consider the rate of change in volume positive.
b) If the water is flowing out of the tub we would consider the rate of change in volume negative.

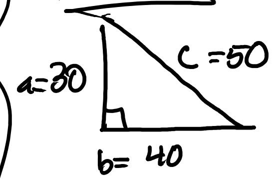
Examples

1. Two people are walking perpendicular to each other toward the same corner at 2ft/second and 4ft/second respectively. At the time when the first person is 30ft from the corner and the second is 40ft from the corner, at what rate is the distance between them changing?



$\frac{da}{dt} = -2\ \text{ft/sec}$ $\quad \frac{db}{dt} = -4\ \text{ft/sec}$

moment:



$\frac{dc}{dt}$

$$a^2 + b^2 = c^2$$

$$2a \cdot \frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

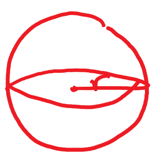
$$2(30)(-2) + 2(40)(-4) = 2(50)\frac{dc}{dt}$$

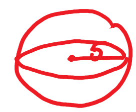
$\frac{dc}{dt} = -4.4\ \text{ft/s}$

2. A spherical balloon is losing volume at a rate of 20π cubic inches/second. How fast is the radius changing at the time the radius is 5 inches?

speed

moment





$\frac{dV}{dt} = -20\pi\ \text{in}^3/\text{sec}$

$\left|\frac{dr}{dt}\right| = ?$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$-20\pi = 4\pi(5)^2\frac{dr}{dt}$$

$$-20\pi = 100\pi\frac{dr}{dt}$$

$\frac{dr}{dt} = -1/5$

$\left|\frac{dr}{dt}\right| = 1/5\ \text{in/sec}$

Related Rates Solution Strategies

1. Draw a picture. Label constant values and assign variables to things that change.
2. Translate the given information in the problem into calculus notation. Do the same thing for what you are asked to find. For example, the area is decreasing at $15\frac{ft^2}{\min}$ *becomes* $\frac{dA}{dt}=-15\frac{ft^2}{\min}$
3. Write a formula/equation relating the variables whose rates of change you are solving for and the variables whose rates of change you are given. Look to geometry for many formulas. Important: At this stage you may substitute for a quantity that is constant; however, don't freeze your problem by substituting a number for a quantity that is changing - keep variables variable!
4. Differentiate implicitly with respect to time. Use all differentiation rules that apply.
5. Now plug in numbers and do calculations. If you round off an answer, use three decimal places.
6. Use complete sentences to answer the question that is asked.