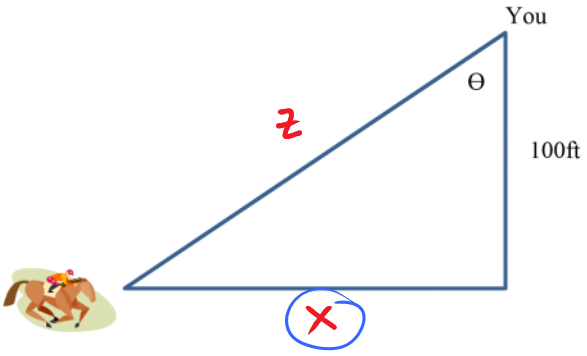
**4.6C Notes – More Related Rates**

1. You are watching a horse race and are seated 100 feet from the track. You are rooting for Charlie Horse who is running at a speed of 59 ft/sec. How fast is the angle changing in the figure below when the racehorse is right in front of you? 2 seconds later?




$\frac{dx}{dt} = 59$ ft/sec $\Big\} \frac{d\theta}{dt} = ?$

$\theta$ = Oradians

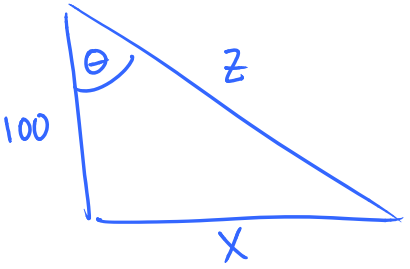
$\tan\theta = \frac{x}{100}$

$100\tan\theta = x$

$100\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{dx}{dt}$

$100(\sec\theta)^2 \frac{d\theta}{dt} = 59$

$100\frac{d\theta}{dt} = 59$ $\quad \frac{d\theta}{dt} = \frac{59}{100}$ rad/sec




b.

$\frac{dx}{dt} = 59$ $\quad 100\sec^2(\theta)\frac{d\theta}{dt} = \frac{dx}{dt}$

$\frac{d\theta}{dt} = ?$ $\quad 100(\sec .868)^2 \frac{d\theta}{dt} = 59$

$100\left(\left(\frac{1}{\cos .868}\right)^2\right)\frac{d\theta}{dt} = 59$

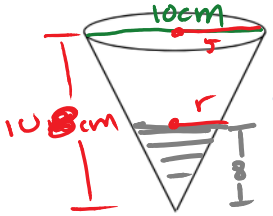

$\tan\theta = \frac{118}{100}$

$\theta = \tan^{-1}\left(\frac{118}{100}\right)$

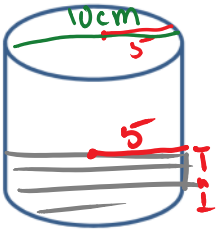
$\theta \approx .868$

$\frac{d\theta}{dt} \approx .246$ rad/sec

2. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 20 cubic cm per min. The diameter of the cone and cylinder are both 10 cm in length. The height of the cone is also 10 cm. How fast is the level of coffee in the coffeepot rising when the coffee in the cone is 8 cm deep?




$\frac{dV}{dt_c} = -20\ cm^3/min$




$\frac{dV}{dt_p} = 20\ cm^3/min$

b. How fast is the level of the cone falling at that moment?

$\frac{dh}{dt_p} = ?$

$V_p = \pi r^2 \cdot h$

$V_p = 25\pi h$

$\frac{dV_p}{dt} = 25\pi \frac{dh}{dt}$

$20 = 25\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{4}{(5\pi)} \approx .255\ cm/min$

b. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 \cdot h$

$= \frac{h^3}{12}\pi$

moment

$h = 8\ cm$

$\frac{dh}{dt} = ?$

$\frac{r}{h} = \frac{5}{10}$

$\frac{r}{h} = \frac{1}{2}$ $\quad r = \frac{1}{2}h$

$\frac{dV}{dt} = \frac{h^2}{4}\pi \frac{dh}{dt}$

$-20 = \frac{64}{4}\pi \frac{dh}{dt}$

$-20 = 16\pi \frac{dh}{dt}$

$\frac{-5}{4\pi} = \frac{dh}{dt}$

$\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$

$\frac{dr}{dt} = \frac{1}{2}\left(\frac{5}{4\pi}\right)$

How fast: $\frac{5}{4\pi}\ cm/min$