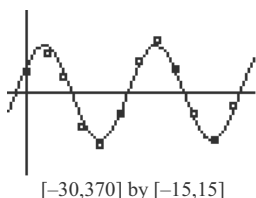


- (c) Using the formula $L2 = Y1(L1)$ (where $Y1$ is the sine regression curve), the residual list is $\{3.73, 7.48, 3.05, -6.50, -9.77, -3.67, 5.63, 9.79, 4.5, -3.66, -8.62, -2.39\}$.
- (d) The following is a scatter plot of the days past January 1 as x -coordinates ($L1$) and the residuals (the differences between the actual number of minutes ($L2$) and the number of minutes predicted by the regression line ($Y1$)) as y -coordinates ($L3$) for the time of day that the sunset occurred in Honolulu in 2009. The sine regression curve is $y = 8.73 \sin(10.034x + 0.622) - 0.05$. (Note: Round $L3$ to 2 decimal places to obtain this answer.) This is another fairly good fit, which indicates that the residuals are not due to chance. There is a periodic variation that is most probably due to physical causes.



- (e) The first regression indicates that the data are periodic and nearly sinusoidal. The second regression indicates that the *variation* of the data around the predicted values is also periodic and nearly sinusoidal. Periodic variation around periodic models is a predictable consequence of bodies orbiting bodies, but ancient astronomers had a difficult time reconciling the data with their simpler models of the universe.

Section 5.5 The Law of Sines

Exploration 1

- If $BC \leq AB$, the segment will not reach from point B to the dotted line. On the other hand, if $BC > AB$, then a circle of radius BC will intersect the dotted line in a unique point. (Note that the line only extends to the left of point A .)
- A circle of radius BC will be tangent to the dotted line at C if $BC = h$, thus determining a unique triangle. It will miss the dotted line entirely if $BC < h$, thus determining zero triangles.
- The second point (C_2) is the reflection of the first point (C_1) on the other side of the altitude.
- $\sin C_2 = \sin(\pi - C_1) = \sin \pi \cos C_1 - \cos \pi \sin C_1 = \sin C_1$.
- If $BC \geq AB$, then BC can only extend to the right of the altitude, thus determining a unique triangle.

Quick Review 5.5

- $a = bc/d$
- $b = ad/c$
- $c = ad/b$
- $d = bc/a$

- $\frac{7 \sin 48^\circ}{\sin 23^\circ} \approx 13.314$
- $\frac{9 \sin 121^\circ}{\sin 14^\circ} \approx 31.888$
- $x = \sin^{-1} 0.3 \approx 17.458^\circ$
- $x = 180^\circ - \sin^{-1} 0.3 \approx 162.542^\circ$
- $x = 180^\circ - \sin^{-1}(-0.7) \approx 224.427^\circ$
- $x = 360^\circ + \sin^{-1}(-0.7) \approx 315.573^\circ$

Section 5.5 Exercises

- Given: $b = 3.7, B = 45^\circ, A = 60^\circ$ — an AAS case.
 $C = 180^\circ - (A + B) = 75^\circ$;
 $\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow a = \frac{b \sin A}{\sin B} = \frac{3.7 \sin 60^\circ}{\sin 45^\circ} \approx 4.5$;
 $\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow c = \frac{b \sin C}{\sin B} = \frac{3.7 \sin 75^\circ}{\sin 45^\circ} \approx 5.1$
- Given: $c = 17, B = 15^\circ, C = 120^\circ$ — an AAS case.
 $A = 180^\circ - (B + C) = 45^\circ$;
 $\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow a = \frac{c \sin A}{\sin C} = \frac{17 \sin 45^\circ}{\sin 120^\circ} \approx 13.9$;
 $\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow b = \frac{c \sin B}{\sin C} = \frac{17 \sin 15^\circ}{\sin 120^\circ} \approx 5.1$
- Given: $A = 100^\circ, C = 35^\circ, a = 22$ — an AAS case.
 $B = 180^\circ - (A + C) = 45^\circ$;
 $b = \frac{a \sin B}{\sin A} = \frac{22 \sin 45^\circ}{\sin 100^\circ} \approx 15.8$;
 $c = \frac{a \sin C}{\sin A} = \frac{22 \sin 35^\circ}{\sin 100^\circ} \approx 12.8$
- Given: $A = 81^\circ, B = 40^\circ, b = 92$ — an AAS case.
 $C = 180^\circ - (A + B) = 59^\circ$;
 $a = \frac{b \sin A}{\sin B} = \frac{92 \sin 81^\circ}{\sin 40^\circ} \approx 141.4$;
 $c = \frac{b \sin C}{\sin B} = \frac{92 \sin 59^\circ}{\sin 40^\circ} \approx 122.7$
- Given: $A = 40^\circ, B = 30^\circ, b = 10$ — an AAS case.
 $C = 180^\circ - (A + B) = 110^\circ$;
 $a = \frac{b \sin A}{\sin B} = \frac{10 \sin 40^\circ}{\sin 30^\circ} \approx 12.9$;
 $c = \frac{b \sin C}{\sin B} = \frac{10 \sin 110^\circ}{\sin 30^\circ} \approx 18.8$
- Given: $A = 50^\circ, B = 62^\circ, a = 4$ — an AAS case.
 $C = 180^\circ - (A + B) = 68^\circ$;
 $b = \frac{a \sin B}{\sin A} = \frac{4 \sin 62^\circ}{\sin 50^\circ} \approx 4.6$;
 $c = \frac{a \sin C}{\sin A} = \frac{4 \sin 68^\circ}{\sin 50^\circ} \approx 4.8$
- Given: $A = 33^\circ, B = 70^\circ, b = 7$ — an AAS case.
 $C = 180^\circ - (A + B) = 77^\circ$;
 $a = \frac{b \sin A}{\sin B} = \frac{7 \sin 33^\circ}{\sin 70^\circ} \approx 4.1$;
 $c = \frac{b \sin C}{\sin B} = \frac{7 \sin 77^\circ}{\sin 70^\circ} \approx 7.3$

8. Given: $B = 16^\circ, C = 103^\circ, c = 12$ — an AAS case.

$$A = 180^\circ - (B + C) = 61^\circ;$$

$$a = \frac{c \sin A}{\sin C} = \frac{12 \sin 61^\circ}{\sin 103^\circ} \approx 10.8;$$

$$b = \frac{c \sin B}{\sin C} = \frac{12 \sin 16^\circ}{\sin 103^\circ} \approx 3.4$$

9. Given: $A = 32^\circ, a = 17, b = 11$ — an SSA case.

$$h = b \sin A \approx 5.8; h < b < a, \text{ so there is one triangle.}$$

$$B = \sin^{-1}\left(\frac{b \sin A}{a}\right) = \sin^{-1}(0.342 \dots) \approx 20.1^\circ$$

$$C = 180^\circ - (A + B) \approx 127.9^\circ;$$

$$c = \frac{a \sin C}{\sin A} = \frac{17 \sin 127.9^\circ}{\sin 32^\circ} \approx 25.3$$

10. Given: $A = 49^\circ, a = 32, b = 28$ — an SSA case.

$$h = b \sin A \approx 21.1; h < b < a, \text{ so there is one triangle.}$$

$$B = \sin^{-1}\left(\frac{b \sin A}{a}\right) = \sin^{-1}(0.660 \dots) \approx 41.3^\circ$$

$$C = 180^\circ - (A + B) = 89.7^\circ;$$

$$c = \frac{a \sin C}{\sin A} = \frac{32 \sin 89.7^\circ}{\sin 49^\circ} \approx 42.4$$

11. Given: $B = 70^\circ, b = 14, c = 9$ — an SSA case.

$$h = c \sin B \approx 8.5; h < c < b, \text{ so there is one triangle.}$$

$$C = \sin^{-1}\left(\frac{c \sin B}{b}\right) = \sin^{-1}(0.604 \dots) \approx 37.2^\circ$$

$$A = 180^\circ - (B + C) \approx 72.8^\circ;$$

$$a = \frac{b \sin A}{\sin B} = \frac{14 \sin 72.8^\circ}{\sin 70^\circ} \approx 14.2$$

12. Given: $C = 103^\circ, b = 46, c = 61$ — an SSA case.

$$h = b \sin C \approx 44.8; h < b < c, \text{ so there is one triangle.}$$

$$B = \sin^{-1}\left(\frac{b \sin C}{c}\right) = \sin^{-1}(0.734 \dots) \approx 47.3^\circ$$

$$A = 180^\circ - (B + C) = 29.7^\circ;$$

$$a = \frac{c \sin A}{\sin C} = \frac{61 \sin 29.7^\circ}{\sin 103^\circ} \approx 31.0$$

13. Given: $A = 36^\circ, a = 2, b = 7. h = b \sin A \approx 4.1;$

$$a < h, \text{ so no triangle is formed.}$$

14. Given: $B = 82^\circ, b = 17, c = 15. h = c \sin B \approx 14.9;$

$$h < c < b, \text{ so there is one triangle.}$$

15. Given: $C = 36^\circ, a = 17, c = 16. h = a \sin C \approx 10.0;$

$$h < c < a, \text{ so there are two triangles.}$$

16. Given: $A = 73^\circ, a = 24, b = 28. h = b \sin A \approx 26.8;$

$$a < h, \text{ so no triangle is formed.}$$

17. Given: $C = 30^\circ, a = 18, c = 9. h = a \sin C = 9;$

$$h = c, \text{ so there is one triangle.}$$

18. Given: $B = 88^\circ, b = 14, c = 62. h = c \sin B \approx 62.0;$

$$b < h, \text{ so no triangle is formed.}$$

19. Given: $A = 64^\circ, a = 16, b = 17. h = b \sin A \approx 15.3;$

$$h < a < b, \text{ so there are two triangles.}$$

$$B_1 = \sin^{-1}\left(\frac{b \sin A}{a}\right) = \sin^{-1}(0.954 \dots) \approx 72.7^\circ$$

$$C_1 = 180^\circ - (A + B_1) \approx 43.3^\circ;$$

$$c_1 = \frac{a \sin C_1}{\sin A} = \frac{16 \sin 43.3^\circ}{\sin 64^\circ} \approx 12.2$$

Or (with B obtuse):

$$B_2 = 180^\circ - B_1 \approx 107.3^\circ;$$

$$C_2 = 180^\circ - (A + B_2) \approx 8.7^\circ;$$

$$c_2 = \frac{a \sin C_2}{\sin A} \approx 2.7$$

20. Given: $B = 38^\circ, b = 21, c = 25. h = c \sin B \approx 15.4;$
 $h < b < c$, so there are two triangles.

$$C_1 = \sin^{-1}\left(\frac{c \sin B}{b}\right) = \sin^{-1}(0.732 \dots) \approx 47.1^\circ;$$

$$A_1 = 180^\circ - (B + C_1) \approx 94.9^\circ;$$

$$a_1 = \frac{b \sin A_1}{\sin B} = \frac{21 \sin 94.9^\circ}{\sin 38^\circ} \approx 34.0$$

Or (with C obtuse):

$$C_2 = 180^\circ - C_1 \approx 132.9^\circ;$$

$$A_2 = 180^\circ - (B + C_2) \approx 9.1^\circ;$$

$$a_2 = \frac{b \sin A_2}{\sin B} \approx 5.4$$

21. Given: $C = 68^\circ, a = 19, c = 18. h = a \sin C \approx 17.6;$
 $h < c < a$, so there are two triangles.

$$A_1 = \sin^{-1}\left(\frac{a \sin C}{c}\right) = \sin^{-1}(0.978 \dots) \approx 78.2^\circ;$$

$$B_1 = 180^\circ - (A + C) \approx 33.8^\circ;$$

$$b_1 = \frac{c \sin B_1}{\sin C} = \frac{18 \sin 33.8^\circ}{\sin 68^\circ} \approx 10.8$$

Or (with A obtuse):

$$A_2 = 180^\circ - A_1 \approx 101.8^\circ;$$

$$B_2 = 180^\circ - (A_2 + C) \approx 10.2^\circ;$$

$$b_2 = \frac{c \sin B_2}{\sin C} \approx 3.4$$

22. Given: $B = 57^\circ, a = 11, b = 10. h = a \sin B \approx 9.2;$
 $h < b < a$, so there are two triangles.

$$A_1 = \sin^{-1}\left(\frac{a \sin B}{b}\right) = \sin^{-1}(0.922 \dots) \approx 67.3^\circ;$$

$$C_1 = 180^\circ - (A_1 + B) \approx 55.7^\circ;$$

$$c_1 = \frac{b \sin C_1}{\sin B} = \frac{10 \sin 55.7^\circ}{\sin 57^\circ} \approx 9.9$$

Or (with A obtuse):

$$A_2 = 180^\circ - A_1 \approx 112.7^\circ;$$

$$C_2 = 180^\circ - (A_2 + B) \approx 10.3^\circ;$$

$$c_2 = \frac{b \sin C_2}{\sin B} \approx 2.1$$

23. $h = 10 \sin 42^\circ \approx 6.69$, so:

(a) $6.69 < b < 10$.

(b) $b \approx 6.69$ or $b \geq 10$.

(c) $b < 6.69$.

24. $h = 12 \sin 53^\circ \approx 9.58$, so:

(a) $9.58 < c < 12$.

(b) $c \approx 9.58$ or $c \geq 12$.

(c) $c < 9.58$.

25. (a) No: This is an SAS case.

(b) No: Only two pieces of information given.

26. (a) Yes: This is an AAS case.

$$B = 180^\circ - (A + C) = 32^\circ;$$

$$b = \frac{a \sin B}{\sin A} = \frac{81 \sin 32^\circ}{\sin 29^\circ} \approx 88.5;$$

$$c = \frac{a \sin C}{\sin A} = \frac{81 \sin 119^\circ}{\sin 29^\circ} \approx 146.1$$

(b) No: This is an SAS case.

27. Given: $A = 61^\circ$, $a = 8$, $b = 21$ — an SSA case.

$$h = b \sin A = 18.4; a < h, \text{ so no triangle is formed.}$$

28. Given: $B = 47^\circ$, $a = 8$, $b = 21$ — an SSA case.

$$h = a \sin B \approx 5.9; h < a < b, \text{ so there is one triangle.}$$

$$A = \sin^{-1}\left(\frac{a \sin B}{b}\right) = \sin^{-1}(0.278\dots) \approx 16.2^\circ;$$

$$C = 180^\circ - (A + B) = 116.8^\circ;$$

$$c = \frac{b \sin C}{\sin B} = \frac{21 \sin 116.8^\circ}{\sin 47^\circ} \approx 25.6$$

29. Given: $A = 136^\circ$, $a = 15$, $b = 28$ — an SSA case.

$$h = b \sin A \approx 19.5; a < h, \text{ so no triangle is formed.}$$

30. Given: $C = 115^\circ$, $b = 12$, $c = 7$ — an SSA case.

$$h = b \sin C \approx 10.9; c < h, \text{ so no triangle is formed.}$$

31. Given: $B = 42^\circ$, $c = 18$, $C = 39^\circ$ — an AAS case.

$$A = 180^\circ - (B + C) = 99^\circ;$$

$$a = \frac{c \sin A}{\sin C} = \frac{18 \sin 99^\circ}{\sin 39^\circ} \approx 28.3;$$

$$b = \frac{c \sin B}{\sin C} = \frac{18 \sin 42^\circ}{\sin 39^\circ} \approx 19.1$$

32. Given: $A = 19^\circ$, $b = 22$, $B = 47^\circ$ — an AAS case.

$$C = 180^\circ - (A + B) = 114^\circ;$$

$$a = \frac{b \sin A}{\sin B} = \frac{22 \sin 19^\circ}{\sin 47^\circ} \approx 9.8;$$

$$c = \frac{b \sin C}{\sin B} = \frac{22 \sin 114^\circ}{\sin 47^\circ} \approx 27.5$$

33. Given: $C = 75^\circ$, $b = 49$, $c = 48$. — an SSA case.

$$h = b \sin C \approx 47.3; h < c < b, \text{ so there are two triangles.}$$

$$B_1 = \sin^{-1}\left(\frac{b \sin C}{c}\right) = \sin^{-1}(0.986\dots) \approx 80.4^\circ;$$

$$A_1 = 180^\circ - (B_1 + C) \approx 24.6^\circ;$$

$$a_1 = \frac{c \sin A_1}{\sin C} = \frac{48 \sin 24.6^\circ}{\sin 75^\circ} \approx 20.7$$

Or (with B obtuse):

$$B_2 = 180^\circ - B_1 \approx 99.6^\circ;$$

$$A_2 = 180^\circ - (B_2 + C) \approx 5.4^\circ;$$

$$a_2 = \frac{c \sin A_2}{\sin C} \approx 4.7$$

34. Given: $A = 54^\circ$, $a = 13$, $b = 15$. — an SSA case.

$$h = b \sin A \approx 12.1; h < a < b, \text{ so there are two triangles.}$$

$$B_1 = \sin^{-1}\left(\frac{b \sin A}{a}\right) = \sin^{-1}(0.933\dots) \approx 69.0^\circ$$

$$C_1 = 180^\circ - (A + B_1) \approx 57.0^\circ;$$

$$c_1 = \frac{a \sin C_1}{\sin A} = \frac{13 \sin 57.0^\circ}{\sin 54^\circ} \approx 13.5$$

Or (with B obtuse):

$$B_2 = 180^\circ - B_1 \approx 111.0^\circ;$$

$$C_2 = 180^\circ - (A + B_2) \approx 15.0^\circ;$$

$$c_2 = \frac{a \sin C_2}{\sin A} \approx 4.2$$

35. Cannot be solved by law of sines (an SAS case).

36. Cannot be solved by law of sines (an SAS case).

37. Given: $c = AB = 56$, $A = 72^\circ$, $B = 53^\circ$ — an ASA case, so $C = 180^\circ - (A + B) = 55^\circ$

$$(a) AC = b = \frac{c \sin B}{\sin C} = \frac{56 \sin 53^\circ}{\sin 55^\circ} \approx 54.6 \text{ ft.}$$

$$(b) h = b \sin A = a \sin B \approx 51.9 \text{ ft.}$$

38. Given: $c = 25$, $A = 90^\circ - 38^\circ = 52^\circ$,

$B = 90^\circ - 53^\circ = 37^\circ$ — an ASA case, so

$$C = 180^\circ - (A + B) = 91^\circ \text{ and}$$

$$a = \frac{c \sin A}{\sin C} = \frac{25 \sin 52^\circ}{\sin 91^\circ} \approx 19.7 \text{ mi.}$$

$$b = \frac{c \sin B}{\sin C} = \frac{25 \sin 37^\circ}{\sin 91^\circ} \approx 15.0 \text{ mi,}$$

and finally $h = b \sin A = a \sin B \approx 11.9 \text{ mi.}$

39. Given: $c = 16$, $C = 90^\circ - 62 = 28^\circ$,

$B = 90^\circ + 15^\circ = 105^\circ$ — an AAS case.

$A = 180^\circ - (B + C) = 47^\circ$, so

$$a = \frac{c \sin A}{\sin C} = \frac{16 \sin 47^\circ}{\sin 28^\circ} \approx 24.9 \text{ ft.}$$

40. Given: $c = 2.32$, $A = 28^\circ$, $B = 37^\circ$ — an ASA case.

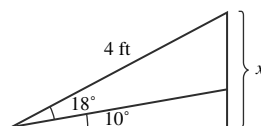
$$C = 180^\circ - (A + B) = 115^\circ;$$

$$a = \frac{c \sin A}{\sin C} = \frac{2.32 \sin 28^\circ}{\sin 115^\circ} \approx 1.2 \text{ mi.}$$

$$b = \frac{c \sin B}{\sin C} = \frac{2.32 \sin 37^\circ}{\sin 115^\circ} \approx 1.5 \text{ mi.}$$

Therefore, the altitude is $h = b \sin A \approx (1.5) \sin 28^\circ \approx 0.7 \text{ mi}$ — or $a \sin B \approx (1.2) \sin 37^\circ \text{ mi} \approx 0.7 \text{ mi.}$

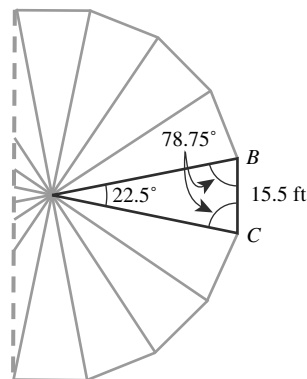
41.



The length of the brace is the leg of the larger triangle.

$$\sin 28^\circ = \frac{x}{4}, \text{ so } x = 1.9 \text{ ft.}$$

42.



The center of the wheel (A) and two adjacent chairs

(B and C) form a triangle with $a = 15.5$, $A = \frac{360^\circ}{16}$

$= 22.5^\circ$, and $B = C = 78.75^\circ$. This is an ASA case, so the radius is $b = c = \frac{a \sin B}{\sin A} = \frac{15.5 \sin 78.75^\circ}{\sin 22.5^\circ} \approx 39.7 \text{ ft.}$

Alternatively, let D be the midpoint of \overline{BC} , and consider right $\triangle ABD$, with $m\angle BAD = 11.25^\circ$ and $BD = 7.75$ ft; then r is the hypotenuse of this triangle, so

$$r = \frac{7.75}{\sin 11.25^\circ} \approx 39.7 \text{ ft.}$$

43. Consider the triangle with vertices at the top of the flagpole (A) and the two observers (B and C). Then $a = 600$, $B = 19^\circ$, and $C = 21^\circ$ (an ASA case), so

$$A = 180^\circ - (B + C) = 140^\circ;$$

$$b = \frac{a \sin B}{\sin A} = \frac{600 \sin 19^\circ}{\sin 140^\circ} \approx 303.9;$$

$$c = \frac{a \sin C}{\sin A} = \frac{600 \sin 21^\circ}{\sin 140^\circ} \approx 334.5;$$

and finally $h = b \sin C = c \sin B \approx 108.9$ ft.

44. Consider the triangle with vertices at the top of the tree (A) and the two observers (B and C). Then $a = 400$, $B = 15^\circ$, and $C = 20^\circ$ (an ASA case), so

$$A = 180^\circ - (B + C) = 145^\circ;$$

$$b = \frac{a \sin B}{\sin A} = \frac{400 \sin 15^\circ}{\sin 145^\circ} \approx 180.5;$$

$$c = \frac{a \sin C}{\sin A} = \frac{400 \sin 20^\circ}{\sin 145^\circ} \approx 238.5;$$

and finally $h = b \sin C = c \sin B \approx 61.7$ ft.

45. Given: $c = 20$, $B = 52^\circ$, $C = 33^\circ$ — an AAS case.

$$A = 180^\circ - (B + C) = 95^\circ, \text{ so}$$

$$a = \frac{c \sin A}{\sin C} = \frac{20 \sin 95^\circ}{\sin 33^\circ} \approx 36.6 \text{ mi, and}$$

$$b = \frac{c \sin B}{\sin C} = \frac{20 \sin 52^\circ}{\sin 33^\circ} \approx 28.9 \text{ mi.}$$

46. We use the mean (average) measurements for A , B , and AB , which are 79.7° , 83.9° , and 25.9 feet, respectively. This gives 16.4° for angle C . By the law of sines,

$$AC = \frac{25.9 \sin 83.9^\circ}{\sin 16.4^\circ} \approx 91.2 \text{ ft.}$$

47. True. By the law of sines, $\frac{\sin A}{a} = \frac{\sin B}{b}$,

which is equivalent to $\frac{\sin A}{\sin B} = \frac{a}{b}$ (since $\sin A, \sin B \neq 0$).

48. False. By the law of sines, the third side of the triangle measures $\frac{10 \sin 100^\circ}{\sin 40^\circ}$, which is about 15.32 inches. That makes the perimeter about $10 + 10 + 15.32 = 35.32$, which is less than 36 inches.

49. The third angle is 32° . By the law of sines, $\frac{\sin 32^\circ}{12.0} = \frac{\sin 53^\circ}{x}$, which can be solved for x .

The answer is C.

50. With SSA, the known side opposite the known angle sometimes has two different possible positions. The answer is D.

51. The longest side is opposite the largest angle, while the shortest side is opposite the smallest angle. By the law of sines, $\frac{\sin 50^\circ}{9.0} = \frac{\sin 70^\circ}{x}$, which can be solved for x .

The answer is A.

52. Because $BC > AB$, only one triangle is possible. The answer is B.

53. (a) Given any triangle with side lengths a , b , and c , the law of sines says that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

But we can also find another triangle (using ASA) with two angles the same as the first (in which case the third angle is also the same) and a different side length — say, a' . Suppose that $a' = ka$ for some constant k . Then for this new triangle, we have $\frac{\sin A}{a'} = \frac{\sin B}{b'} = \frac{\sin C}{c'}$. Since $\frac{\sin A}{a'} = \frac{\sin A}{ka} = \frac{1}{k} \cdot \frac{\sin A}{a}$, we can see that $\frac{\sin B}{b'} = \frac{1}{k} \cdot \frac{\sin B}{b}$,

so that $b' = kb$ and similarly, $c' = kc$. So for any choice of a positive constant k , we can create a triangle with angles A , B , and C .

- (b) Possible answers: $a = 1$, $b = \sqrt{3}$, $c = 2$ (or any set of three numbers proportional to these).
(c) Any set of three identical numbers.
54. In each proof, assume that sides a , b , and c are opposite angles A , B , and C , and that c is the hypotenuse.

(a) $\frac{\sin A}{a} = \frac{\sin 90^\circ}{c}$

$$\frac{\sin A}{a} = \frac{1}{c}$$

$$\sin A = \frac{a}{c} = \frac{\text{opp}}{\text{hyp}}$$

(b) $\frac{\sin B}{b} = \frac{\sin 90^\circ}{c}$

$$\frac{\cos(\pi/2 - B)}{b} = \frac{1}{c}$$

$$\cos A = \frac{b}{c} = \frac{\text{adj}}{\text{hyp}}$$

(c) $\frac{\sin A}{a} = \frac{\sin B}{b}$

$$\frac{\sin A}{\sin B} = \frac{a}{b}$$

$$\frac{\sin A}{\cos A} = \frac{a}{b}$$

$$\tan A = \frac{a}{b} = \frac{\text{opp}}{\text{adj}}$$

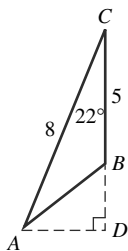
55. (a) $h = AB \sin A$

(b) $BC < AB \sin A$

(c) $BC \geq AB$ or $BC = AB \sin A$

(d) $AB \sin A < BC < AB$

56. Drawing the line suggested in the hint, and extending \overline{BC} to meet that line at, say, D , gives right $\triangle ADC$ and right $\triangle ADB$.



Then $AD = 8 \sin 22^\circ \approx 3.0$ and $DC = 8 \cos 22^\circ \approx 7.4$, so $DB = DC - 5$ and $c = AB = \sqrt{AD^2 + DB^2} \approx 3.9$.

Finally,

$$A = (90^\circ - 22^\circ) - \sin^{-1}\left(\frac{DB}{AB}\right) \approx 29.1^\circ \text{ and}$$

$$B = 180^\circ - A - C \approx 128.9^\circ.$$

57. Given: $c = 4.1$, $B = 25^\circ$, $C = 36.5^\circ - 25^\circ = 11.5^\circ$. An AAS case: $A = 180^\circ - (B + C) = 143.5^\circ$, so

$$AC = b = \frac{c \sin B}{\sin C} = \frac{4.1 \sin 25^\circ}{\sin 11.5^\circ} \approx 8.7 \text{ mi, and}$$

$$BC = a = \frac{c \sin A}{\sin C} = \frac{4.1 \sin 143.5^\circ}{\sin 11.5^\circ} \approx 12.2 \text{ mi.}$$

The height is $h = a \sin 25^\circ = b \sin 36.5^\circ \approx 5.2$ mi.

Section 5.6 The Law of Cosines

Exploration 1

1. The semiperimeters are 154 and 150.

$$A = \sqrt{154(154 - 115)(154 - 81)(154 - 112)} \\ + \sqrt{150(150 - 112)(150 - 102)(150 - 86)} \\ = 8475.742818 \text{ paces}^2$$

2. 41,022.59524 square feet

3. 0.0014714831 square miles

4. 0.94175 acres

5. The estimate of "a little over an acre" seems questionable, but the roughness of their measurement system does not provide firm evidence that it is incorrect. If Jim and Barbara wish to make an issue of it with the owner, they would be well advised to get some more reliable data.

6. Yes. In fact, any polygonal region can be subdivided into triangles.

Quick Review 5.6

1. $A = \cos^{-1}\left(\frac{3}{5}\right) \approx 53.130^\circ$

2. $C = \cos^{-1}(-0.23) \approx 103.297^\circ$

3. $A = \cos^{-1}(-0.68) \approx 132.844^\circ$

4. $C = \cos^{-1}\left(\frac{1.92}{3}\right) \approx 50.208^\circ$

5. (a) $\cos A = \frac{81 - x^2 - y^2}{-2xy} = \frac{x^2 + y^2 - 81}{2xy}$

(b) $A = \cos^{-1}\left(\frac{x^2 + y^2 - 81}{2xy}\right)$

6. (a) $\cos A = \frac{y^2 - x^2 - 25}{-10} = \frac{x^2 - y^2 + 25}{10}$

(b) $A = \cos^{-1}\left(\frac{x^2 - y^2 + 25}{10}\right)$

7. One answer: $(x - 1)(x - 2) = x^2 - 3x + 2$.
Generally: $(x - a)(x - b) = x^2 - (a + b)x + ab$
for any two positive numbers a and b .

8. One answer: $(x - 1)(x + 1) = x^2 - 1$.
Generally: $(x - a)(x + b) = x^2 - (a - b)x - ab$
for any two positive numbers a and b .

9. One answer: $(x - i)(x + i) = x^2 + 1$.

10. One answer: $(x - 1)^2 = x^2 - 2x + 1$.
Generally: $(x - a)^2 = x^2 - 2ax + a^2$
for any positive number a .

Section 5.6 Exercises

1. Given: $B = 131^\circ$, $c = 8$, $a = 13$ — an SAS case.

$$b = \sqrt{a^2 + c^2 - 2ac \cos B} \approx \sqrt{369.460} \approx 19.2;$$

$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right) \approx \cos^{-1}(0.949) \approx 18.3^\circ;$$

$$A = 180^\circ - (B + C) \approx 30.7^\circ.$$

2. Given: $C = 42^\circ$, $b = 12$, $a = 14$ — an SAS case.

$$c = \sqrt{a^2 + b^2 - 2ab \cos C} \approx \sqrt{90.303} \approx 9.5;$$

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.167) \approx 80.3^\circ;$$

$$B = 180^\circ - (A + C) \approx 57.7^\circ.$$

3. Given: $a = 27$, $b = 19$, $c = 24$ — an SSS case.

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.228) \approx 76.8^\circ;$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.728) \approx 43.2^\circ;$$

$$C = 180^\circ - (A + B) \approx 60^\circ.$$

4. Given: $a = 28$, $b = 35$, $c = 17$ — an SSS case.

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.613) \approx 52.2^\circ;$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.159) \approx 99.2^\circ;$$

$$C = 180^\circ - (A + B) \approx 28.6^\circ.$$

5. Given: $A = 55^\circ$, $b = 12$, $c = 7$ — an SAS case.

$$a = \sqrt{b^2 + c^2 - 2bc \cos A} \approx \sqrt{96.639} \approx 9.8;$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.011) \approx 89.3^\circ;$$

$$C = 180^\circ - (A + B) \approx 35.7^\circ.$$

6. Given: $B = 35^\circ$, $a = 43$, $c = 19$ — an SAS case.

$$b = \sqrt{a^2 + c^2 - 2ac \cos B} \approx \sqrt{871.505} \approx 29.5;$$

$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right) \approx \cos^{-1}(0.929) \approx 21.7^\circ;$$

$$A = 180^\circ - (B + C) \approx 123.3^\circ.$$

7. Given: $a = 12$, $b = 21$, $C = 95^\circ$ — an SAS case.

$$c = \sqrt{a^2 + b^2 - 2ab \cos C} \approx \sqrt{628.926} \approx 25.1;$$

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.879) \approx 28.5^\circ;$$

$$B = 180^\circ - (A + C) \approx 56.5^\circ.$$

8. Given: $b = 22, c = 31, A = 82^\circ$ — an SAS case.

$$a = \sqrt{b^2 + c^2 - 2bc \cos A} \approx \sqrt{1255.167} \approx 35.4;$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.788) \approx 37.9^\circ;$$

$$C = 180^\circ - (A + B) \approx 60.1^\circ.$$

9. No triangles possible ($a + c = b$).

10. No triangles possible ($a + b < c$).

11. Given: $a = 3.2, b = 7.6, c = 6.4$ — an SSS case.

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.909) \approx 24.6^\circ;$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.160) \approx 99.2^\circ;$$

$$C = 180^\circ - (A + B) \approx 56.2^\circ.$$

12. No triangles possible ($a + b < c$).

Exercises #13–16 are SSA cases, and can be solved with either the law of sines or the law of cosines. The law of cosines solution is shown.

13. Given: $A = 42^\circ, a = 7, b = 10$ — an SSA case. Solve the quadratic equation $7^2 = 10^2 + c^2 - 2(10)c \cos 42^\circ$, or $c^2 - (14.862\dots)c + 51 = 0$; there are two positive solutions: ≈ 9.487 or 5.376 . Since $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$:

$$c_1 \approx 9.5, B_1 \approx \cos^{-1}(0.294) \approx 72.9^\circ, \text{ and}$$

$$C_1 = 180^\circ - (A + B_1) \approx 65.1^\circ,$$

or

$$c_2 \approx 5.4, B_2 \approx \cos^{-1}(-0.294) \approx 107.1^\circ, \text{ and}$$

$$C_2 = 180^\circ - (A + B_2) \approx 30.9^\circ.$$

14. Given: $A = 57^\circ, a = 11, b = 10$ — an SSA case. Solve the quadratic equation $11^2 = 10^2 + c^2 - 2(10)c \cos 57^\circ$, or $c^2 - (10.893)c - 21 = 0$; there is one positive solution $c = 12.6$. Since $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$,

$$B \approx \cos^{-1}(0.647) \approx 49.7^\circ \text{ and } C = 180^\circ - (A + B) \approx 73.3^\circ.$$

15. Given: $A = 63^\circ, a = 8.6, b = 11.1$ — an SSA case. Solve the quadratic equation $8.6^2 = 11.1^2 + c^2 - 2(11.1)c \cos 63^\circ$, or $c^2 - (10.079)c + 49.25 = 0$; there are no real solutions, so there is no triangle.

16. Given: $A = 71^\circ, a = 9.3, b = 8.5$ — an SSA case. Solve the quadratic equation

$$9.3^2 = 8.5^2 + c^2 - 2(8.5)c \cos 71^\circ, \text{ or}$$

$$c^2 - (5.535)c - 14.24 = 0; \text{ there is one positive}$$

$$\text{solution: } c \approx 7.4. \text{ Since } \cos B = \frac{a^2 + c^2 - b^2}{2ac},$$

$$B \approx \cos^{-1}(0.503) \approx 59.8^\circ \text{ and } C = 180^\circ - (A + B) \approx 49.2^\circ.$$

17. Given: $A = 47^\circ, b = 32, c = 19$ — an SAS case.

$$a = \sqrt{b^2 + c^2 - 2bc \cos A} \approx \sqrt{555.689} \approx 23.573,$$

$$\text{so Area} \approx \sqrt{49431.307} \approx 222.33 \text{ ft}^2 \text{ (using Heron's}$$

$$\text{formula). Or, use Area} = \frac{1}{2}bc \sin A = \frac{1}{2}(32)(19)$$

$$\sin 47^\circ \approx 222.33 \text{ ft}^2.$$

18. Given: $A = 52^\circ, b = 14, c = 21$ — an SAS case.

$$a = \sqrt{b^2 + c^2 - 2bc \cos A} \approx \sqrt{274.991} \approx 16.583,$$

$$\text{so Area} \approx \sqrt{13418.345} \approx 115.84 \text{ m}^2 \text{ (using Heron's}$$

$$\text{formula). Or, use Area} = \frac{1}{2}bc \sin A = \frac{1}{2}(14)(21)$$

$$\sin 52^\circ \approx 115.84 \text{ m}^2.$$

19. Given: $B = 101^\circ, a = 10, c = 22$ — an SAS case.

$$b = \sqrt{a^2 + c^2 - 2ac \cos B} \approx \sqrt{667.955} \approx 25.845,$$

$$\text{so Area} \approx \sqrt{11659.462} \approx 107.98 \text{ cm}^2 \text{ (using Heron's}$$

$$\text{formula). Or, use Area} = \frac{1}{2}ac \sin B = \frac{1}{2}(10)(22)$$

$$\sin 101^\circ \approx 107.98 \text{ cm}^2.$$

20. Given: $C = 112^\circ, a = 1.8, b = 5.1$ — an SAS case.

$$c = \sqrt{a^2 + b^2 - 2ab \cos C} \approx \sqrt{36.128} \approx 6.011,$$

$$\text{so Area} \approx \sqrt{18.111} \approx 4.26 \text{ in}^2 \text{ (using Heron's}$$

$$\text{formula). Or, use Area} = \frac{1}{2}ab \sin C = \frac{1}{2}(1.8)(5.1)$$

$$\sin 112^\circ \approx 4.26 \text{ in}^2.$$

For #21–28, a triangle can be formed if $a + b < c, a + c < b$, and $b + c < a$.

21. $s = \frac{17}{2}$; Area = $\sqrt{66.9375} \approx 8.18$.

22. $s = \frac{21}{2}$; Area = $\sqrt{303.1875} \approx 17.41$.

23. No triangle is formed ($a + b = c$).

24. $s = 27$; Area = $\sqrt{12,960} = 36\sqrt{10} \approx 113.84$.

25. $a = 36.4$; Area = $\sqrt{46,720.3464} \approx 216.15$.

26. No triangle is formed ($a + b < c$).

27. $s = 42.1$; Area = $\sqrt{98,629.1856} \approx 314.05$.

28. $s = 23.8$; Area = $\sqrt{10,269.224} \approx 101.34$.

29. Let $a = 4, b = 5$, and $c = 6$. The largest angle is opposite the largest side, so we call it C . Since $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$, $C = \cos^{-1}\left(\frac{1}{8}\right) \approx 82.819^\circ \approx 1.445$ radians.

30. The shorter diagonal splits the parallelogram into two (congruent) triangles with $a = 26, B = 39^\circ$, and $c = 18$.

$$\text{The diagonal has length } b = \sqrt{a^2 + c^2 - 2ac \cos B}$$

$$\approx \sqrt{272.591} \approx 16.5 \text{ ft.}$$

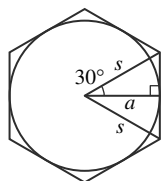
31. Following the method of Example 3, divide the hexagon into six triangles. Each has two 12-inch sides that form a 60° angle.

$$6 \times \frac{1}{2}(12)(12)\sin 60^\circ = 216\sqrt{3} \approx 374.1 \text{ square inches.}$$

32. Following the method of Example 3, divide the nonagon into nine triangles. Each has two 10-inch sides that form a 40° angle.

$$9 \times \frac{1}{2}(10)(10)\sin 40^\circ \approx 289.3 \text{ square inches.}$$

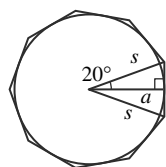
33.



In the figure, $a = 12$ and so $s = 12 \sec 30^\circ = 8\sqrt{3}$.
The area of the hexagon is

$$6 \times \frac{1}{2}(8\sqrt{3})(8\sqrt{3}) \sin 60^\circ = 288\sqrt{3} \\ \approx 498.8 \text{ square inches.}$$

34.



In the figure, $a = 10$ and so $s = 10 \sec 20^\circ$. The area of the nonagon is

$$9 \times \frac{1}{2}(10 \sec 20^\circ)(10 \sec 20^\circ) \sin 40^\circ \approx 327.6 \text{ square inches.}$$

 35. Given: $C = 54^\circ$, $BC = a = 160$, $AC = b = 110$ — an

SAS case. $AB = c = \sqrt{a^2 + b^2 - 2ab \cos C}$
 $\approx \sqrt{17,009.959} \approx 130.42$ ft.

36. (a) The home-to-second segment is the hypotenuse of a right triangle, so the distance from the pitcher's rubber to second base is $90\sqrt{2} - 60.5 \approx 66.8$ ft. This is a bit more than

$$c = \sqrt{60.5^2 + 90^2 - 2(60.5)(90) \cos 45^\circ} \\ \approx \sqrt{4059.857} \approx 63.7 \text{ ft.}$$

(b) $B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.049)$
 $\approx 92.8^\circ$.

37. (a) $c = \sqrt{40^2 + 60^2 - 2(40)(60) \cos 45^\circ}$
 $\approx \sqrt{1805.887} \approx 42.5$ ft.

(b) The home-to-second segment is the hypotenuse of a right triangle, so the distance from the pitcher's rubber to second base is $60\sqrt{2} - 40 \approx 44.9$ ft.

(c) $B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(-0.057)$
 $\approx 93.3^\circ$.

38. Given: $a = 175$, $b = 860$, and $C = 78^\circ$. An SAS case, so
 $AB = c = \sqrt{a^2 + b^2 - 2ab \cos C} \approx \sqrt{707,643.581}$
 ≈ 841.2 ft.

39. (a) Using right $\triangle ACE$, $m\angle CAE = \tan^{-1}\left(\frac{6}{18}\right)$
 $= \tan^{-1}\left(\frac{1}{3}\right) \approx 18.4^\circ$.

(b) Using $A \approx 18.435^\circ$, we have an SAS case, so
 $DF = \sqrt{9^2 + 12^2 - 2(9)(12) \cos A} \approx \sqrt{20.084}$
 ≈ 4.5 ft.

(c) $EF = \sqrt{18^2 + 12^2 - 2(18)(12) \cos A} \approx \sqrt{58.168}$
 ≈ 7.6 ft.

40. After two hours, the planes have traveled 700 and 760 miles, and the angle between them is 22.5° , so the distance is $\sqrt{700^2 + 760^2 - 2(700)(760) \cos 22.5^\circ}$
 $\approx \sqrt{84,592.177} \approx 290.8$ mi.

41. $AB = \sqrt{73^2 + 65^2 - 2(73)(65) \cos 8^\circ}$
 $\approx \sqrt{156.356} \approx 12.5$ yd.

42. $m\angle HAB = 135^\circ$, so

$$HB = \sqrt{20^2 + 20^2 - 2(20)(20) \cos 135^\circ} \\ \approx \sqrt{1365.685} \approx 37.0 \text{ ft.}$$

Note that \overline{AB} is the hypotenuse of an equilateral right triangle with leg length $\frac{20}{\sqrt{2}} = 10\sqrt{2}$, and \overline{HC} is the hypotenuse of an equilateral right triangle with leg length $20 + 10\sqrt{2}$, so $HC = \sqrt{2(20 + 10\sqrt{2})^2} \approx 48.3$ ft.

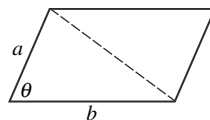
Finally, using right $\triangle HAD$ with leg lengths $HA = 20$ ft and $AD = HC \approx 48.3$ ft, we have
 $HD = \sqrt{HA^2 + AD^2} \approx 52.3$ ft.

43. $AB = c = \sqrt{2^2 + 3^2} = \sqrt{13}$, $AC = b = \sqrt{1^2 + 3^2} = \sqrt{10}$, and $BC = a = \sqrt{1^2 + 2^2} = \sqrt{5}$, so
 $m\angle CAB = A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$
 $= \cos^{-1}\left(\frac{9}{\sqrt{130}}\right) \approx 37.9^\circ$.

44. $\triangle ABC$ is a right triangle ($C = 90^\circ$), with $BC = a = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ and $AC = b = 1$, so $AB = c = \sqrt{a^2 + b^2} = 3$ and $B = m\angle ABC = \sin^{-1}\left(\frac{1}{3}\right) \approx 19.5^\circ$.

45. True. By the law of cosines, $b^2 + c^2 - 2bc \cos A = a^2$, which is a positive number. Since $b^2 + c^2 - 2bc \cos A > 0$, it follows that $b^2 + c^2 > 2bc \cos A$.

46. True. The diagonal opposite angle θ splits the parallelogram into two congruent triangles, each with area $\frac{1}{2} ab \sin \theta$.



47. Following the method of Example 3, divide the dodecagon into 12 triangles. Each has two 12-inch sides that form a 30° angle.

$$12 \times \frac{1}{2}(12)(12) \sin 30^\circ = 432$$

The answer is B.

48. The semiperimeter is $s = (7 + 8 + 9)/2 = 12$. Then by Heron's formula, $A = \sqrt{12(12 - 7)(12 - 8)(12 - 9)} = 12\sqrt{5}$. The answer is B.

49. After 30 minutes, the first boat has traveled 12 miles and the second has traveled 16 miles. By the law of cosines, the two boats are $\sqrt{12^2 + 16^2 - 2(12)(16) \cos 110^\circ} \approx 23.05$ miles apart. The answer is C.

50. By the law of cosines, $12^2 = 17^2 + 25^2 - 2(17)(25)$

$$\cos \theta, \text{ so } \theta = \cos^{-1} \left(\frac{17^2 + 25^2 - 12^2}{2(17)(25)} \right) \approx 25.06^\circ.$$

The answer is E.

51. Consider that an n -sided regular polygon inscribed within a circle can divide into n equilateral triangles, each with equal area of $\frac{r^2}{2} \sin \frac{360^\circ}{n}$. (The two equal sides of the equilateral triangle are of length r , the radius of the circle.) Then, the area of the polygon is exactly $\frac{nr^2}{2} \sin \frac{360^\circ}{n}$.

$$\begin{aligned} 52. \text{ (a) } \frac{b^2 + c^2 - a^2}{2abc} &= \frac{b^2 + c^2 - (b^2 + c^2 - 2bc \cos A)}{2abc} \\ &= \frac{2bc \cos A}{2abc} \\ &= \frac{\cos A}{a} \end{aligned}$$

(b) The identity in (a) has two other equivalent forms:

$$\begin{aligned} \frac{\cos B}{b} &= \frac{a^2 + c^2 - b^2}{2abc} \\ \frac{\cos C}{c} &= \frac{a^2 + b^2 - c^2}{2abc} \end{aligned}$$

We use them all in the proof:

$$\begin{aligned} \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} &= \frac{b^2 + c^2 - a^2}{2abc} + \frac{a^2 + c^2 - b^2}{2abc} + \frac{a^2 + b^2 - c^2}{2abc} \\ &= \frac{b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2}{2abc} \\ &= \frac{a^2 + b^2 + c^2}{2abc} \end{aligned}$$

$$53. \text{ (a) Ship A: } \frac{30.2 - 15.1}{1 \text{ hr}} = 15.1 \text{ knots;}$$

$$\text{Ship B: } \frac{37.2 - 12.4}{2 \text{ hrs}} = 12.4 \text{ knots}$$

$$\begin{aligned} \text{(b) } \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{(15.1)^2 + (12.4)^2 - (8.7)^2}{2(15.1)(12.4)} \end{aligned}$$

$$A = 35.18^\circ$$

$$\begin{aligned} \text{(c) } c^2 &= a^2 + b^2 - 2ab \cos C \\ &= (49.6)^2 + (60.4)^2 - 2(49.6)(60.4) \cos (35.18^\circ) \\ &\approx 1211.04, \text{ so the boats are } 34.8 \text{ nautical miles} \\ &\text{apart at noon.} \end{aligned}$$

54. Use the area formula and the law of sines:

$$\begin{aligned} A_\Delta &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} a \left(\frac{a \sin B}{\sin A} \right) \sin C \quad \left(\text{law of sines} \Rightarrow b = \frac{a \sin B}{\sin A} \right) \\ &= \frac{a^2 \sin B \sin C}{2 \sin A} \end{aligned}$$

55. Let P be the center of the circle. Then,

$$\cos P = \frac{5^2 + 5^2 - 7^2}{2(5)(5)} = 0.02, \text{ so } P \approx 88.9^\circ. \text{ The area}$$

of the segment is $\pi r^2 \cdot \frac{88.9^\circ}{360^\circ} \approx 25\pi \cdot (0.247) \approx 19.39 \text{ in}^2$.

The area of the triangle, however, is $\frac{1}{2}(5)(5) \sin (88.9)$

$\approx 12.50 \text{ in}^2$, so the area of the shaded region is approximately 6.9 in^2 .

Chapter 5 Review

1. $2 \sin 100^\circ \cos 100^\circ = \sin 200^\circ$

2. $\frac{2 \tan 40^\circ}{1 - \tan^2 40^\circ} = \tan 80^\circ$

3. 1; the expression simplifies to $(\cos 2\theta)^2 + (2 \sin \theta \cos \theta)^2 = (\cos 2\theta)^2 + (\sin 2\theta)^2 = 1$.

4. $\cos^2 2x$; the expression can be rewritten $1 - (2 \sin x \cos x)^2 = 1 - (\sin 2x)^2 = \cos^2 2x$.

5. $\cos 3x = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$
 $= (\cos^2 x - \sin^2 x) \cos x - (2 \sin x \cos x) \sin x$
 $= \cos^3 x - 3 \sin^2 x \cos x$
 $= \cos^3 x - 3(1 - \cos^2 x) \cos x$
 $= \cos^3 x - 3 \cos x + 3 \cos^3 x$
 $= 4 \cos^3 x - 3 \cos x$

6. $\cos^2 2x - \cos^2 x = (1 - \sin^2 2x) - (1 - \sin^2 x)$
 $= \sin^2 x - \sin^2 2x$

7. $\tan^2 x - \sin^2 x = \sin^2 x \left(\frac{1 - \cos^2 x}{\cos^2 x} \right)$
 $= \sin^2 x \cdot \frac{\sin^2 x}{\cos^2 x} = \sin^2 x \tan^2 x$

8. $2 \sin \theta \cos^3 \theta + 2 \sin^3 \theta \cos \theta$
 $= (2 \sin \theta \cos \theta)(\cos^2 \theta + \sin^2 \theta)$
 $= (2 \sin \theta \cos \theta)(1) = \sin 2\theta$.

9. $\csc x - \cos x \cot x = \frac{1}{\sin x} - \cos x \cdot \frac{\cos x}{\sin x}$
 $= \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x$

10. $\frac{\tan \theta + \sin \theta}{2 \tan \theta} = \frac{1 + \cos \theta}{2} = \left(\pm \sqrt{\frac{1 + \cos \frac{\theta}{2}}{2}} \right)^2$
 $= \cos^2 \frac{\theta}{2}$

11. Recall that $\tan \theta \cot \theta = 1$. $\frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 + \cot \theta}{1 - \cot \theta}$
 $= \frac{(1 + \tan \theta)(1 - \cot \theta) + (1 + \cot \theta)(1 - \tan \theta)}{(1 - \tan \theta)(1 - \cot \theta)}$
 $= \frac{(1 + \tan \theta - \cot \theta - 1) + (1 + \cot \theta - \tan \theta - 1)}{(1 - \tan \theta)(1 - \cot \theta)}$
 $= \frac{0}{(1 - \tan \theta)(1 - \cot \theta)} = 0$

12. $\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$
 $= 2 \sin \theta \cos^2 \theta + (\cos^2 \theta - \sin^2 \theta) \sin \theta$
 $= 3 \sin \theta \cos^2 \theta - \sin^3 \theta$

13. $\cos^2 \frac{t}{2} = \left[\pm \sqrt{\frac{1 + \cos t}{2}} \right]^2 = \frac{1}{2}(1 + \cos t)$
 $= \left(\frac{1 + \cos t}{2} \right) \left(\frac{\sec t}{\sec t} \right) = \frac{1 + \sec t}{2 \sec t}$

$$\begin{aligned}
 14. \quad & \frac{\tan^3 \gamma - \cot^3 \gamma}{\tan^2 \gamma + \csc^2 \gamma} \\
 &= \frac{(\tan \gamma - \cot \gamma)(\tan^2 \gamma + \tan \gamma \cot \gamma + \cot^2 \gamma)}{\tan^2 \gamma + \csc^2 \gamma} \\
 &= \frac{(\tan \gamma - \cot \gamma)(\tan^2 \gamma + 1 + \cot^2 \gamma)}{\tan^2 \gamma + \csc^2 \gamma} \\
 &= \frac{(\tan \gamma - \cot \gamma)(\tan^2 \gamma + \csc^2 \gamma)}{\tan^2 \gamma + \csc^2 \gamma} = \tan \gamma - \cot \gamma
 \end{aligned}$$

$$\begin{aligned}
 15. \quad & \frac{\cos \phi}{1 - \tan \phi} + \frac{\sin \phi}{1 - \cot \phi} \\
 &= \left(\frac{\cos \phi}{1 - \tan \phi} \right) \left(\frac{\cos \phi}{\cos \phi} \right) + \left(\frac{\sin \phi}{1 - \cot \phi} \right) \left(\frac{\sin \phi}{\sin \phi} \right) \\
 &= \frac{\cos^2 \phi}{\cos \phi - \sin \phi} + \frac{\sin^2 \phi}{\sin \phi - \cos \phi} = \frac{\cos^2 \phi - \sin^2 \phi}{\cos \phi - \sin \phi} \\
 &= \cos \phi + \sin \phi
 \end{aligned}$$

$$\begin{aligned}
 16. \quad & \frac{\cos(-z)}{\sec(-z) + \tan(-z)} = \frac{\cos(z)}{[1 + \sin(-z)]/\cos(z)} \\
 &= \frac{\cos^2(z)}{1 + \sin(-z)} = \frac{1 - \sin^2 z}{1 - \sin z} = 1 + \sin z
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \sqrt{\frac{1 - \cos y}{1 + \cos y}} = \sqrt{\frac{(1 - \cos y)^2}{(1 + \cos y)(1 - \cos y)}} \\
 &= \sqrt{\frac{(1 - \cos y)^2}{1 - \cos^2 y}} = \sqrt{\frac{(1 - \cos y)^2}{\sin^2 y}} \\
 &= \frac{|1 - \cos y|}{|\sin y|} = \frac{1 - \cos y}{|\sin y|}; \text{ since } 1 - \cos y \geq 0, \\
 & \text{we can drop that absolute value.}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \sqrt{\frac{1 - \sin \gamma}{1 + \sin \gamma}} = \sqrt{\frac{(1 - \sin \gamma)(1 + \sin \gamma)}{(1 + \sin \gamma)^2}} \\
 &= \sqrt{\frac{1 - \sin^2 \gamma}{(1 + \sin \gamma)^2}} = \sqrt{\frac{\cos^2 \gamma}{(1 + \sin \gamma)^2}} \\
 &= \frac{|\cos \gamma|}{|1 + \sin \gamma|} = \frac{|\cos \gamma|}{1 + \sin \gamma}; \text{ since } 1 + \sin \gamma \geq 0, \\
 & \text{we can drop that absolute value.}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \tan \left(u + \frac{3\pi}{4} \right) = \frac{\tan u + \tan(3\pi/4)}{1 - \tan u \tan(3\pi/4)} \\
 &= \frac{\tan u + (-1)}{1 - \tan u(-1)} = \frac{\tan u - 1}{1 + \tan u}
 \end{aligned}$$

$$\begin{aligned}
 20. \quad & \frac{1}{4} \sin 4\gamma = \frac{1}{4} \sin 2(2\gamma) = \frac{1}{4} (2 \sin 2\gamma \cos 2\gamma) \\
 &= \frac{1}{2} (2 \sin \gamma \cos \gamma)(\cos^2 \gamma - \sin^2 \gamma) \\
 &= \sin \gamma \cos^3 \gamma - \cos \gamma \sin^3 \gamma
 \end{aligned}$$

$$21. \quad \tan \frac{1}{2} \beta = \frac{1 - \cos \beta}{\sin \beta} = \frac{1}{\sin \beta} - \frac{\cos \beta}{\sin \beta} = \csc \beta - \cot \beta$$

$$\begin{aligned}
 22. \quad & \text{Let } \theta = \arctan t, \text{ so that } \tan \theta = t. \text{ Then} \\
 & \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2t}{1 - t^2}. \text{ Note also that since} \\
 & -1 < t < 1, -\frac{\pi}{4} < \theta < \frac{\pi}{4}, \text{ and therefore } -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}.
 \end{aligned}$$

That means that 2θ is in the range of the arctan function, and so $2\theta = \arctan \frac{2t}{1 - t^2}$, or equivalently

$$\theta = \frac{1}{2} \arctan \frac{2t}{1 - t^2} \text{ — and of course, } \theta = \arctan t.$$

$$\begin{aligned}
 23. \quad & \text{Yes: } \sec x - \sin x \tan x = \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} \\
 &= \frac{1 - \sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x.
 \end{aligned}$$

$$\begin{aligned}
 24. \quad & \text{Yes: } (\sin^2 \alpha - \cos^2 \alpha)(\tan^2 \alpha + 1) \\
 &= (\sin^2 \alpha - \cos^2 \alpha)(\sec^2 \alpha) \\
 &= \frac{\sin^2 \alpha - \cos^2 \alpha}{\cos^2 \alpha} = \frac{\sin^2 \alpha}{\cos^2 \alpha} - 1 = \tan^2 \alpha - 1.
 \end{aligned}$$

$$\begin{aligned}
 25. \quad & \text{Many answers are possible, for example,} \\
 & \sin 3x + \cos 3x \\
 &= (3 \sin x - 4 \sin^3 x) + (4 \cos^3 x - 3 \cos x) \\
 &= 3(\sin x - \cos x) - 4(\sin^3 x - \cos^3 x) \\
 &= (\sin x - \cos x)[3 - 4(\sin^2 x + \sin x \cos x + \cos^2 x)] \\
 &= (\sin x - \cos x)(3 - 4 - 4 \sin x \cos x) \\
 &= (\cos x - \sin x)(1 + 4 \sin x \cos x). \text{ Check} \\
 & \text{other answers with a grapher.}
 \end{aligned}$$

$$\begin{aligned}
 26. \quad & \text{Many answers are possible, for example,} \\
 & \sin 2x + \cos 3x = 2 \sin x \cos x + 4 \cos^3 x - 3 \cos x \\
 &= \cos x(2 \sin x + 4 \cos^2 x - 3) \\
 &= \cos x(2 \sin x + 1 - 4 \sin^2 x). \text{ Check other answers} \\
 & \text{with a grapher.}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad & \text{Many answers are possible, for example,} \\
 & \cos^2 2x - \sin 2x = 1 - \sin^2 2x - \sin 2x \\
 &= 1 - 4 \sin^2 x \cos^2 x - 2 \sin x \cos x. \text{ Check other} \\
 & \text{answers with a grapher.}
 \end{aligned}$$

$$\begin{aligned}
 28. \quad & \text{Many answers are possible, for example (using Review} \\
 & \text{Exercise 12), } \sin 3x - 3 \sin 2x \\
 &= 3 \cos^2 x \sin x - \sin^3 x - 6 \sin x \cos x \\
 &= \sin x(3 \cos^2 x - \sin^2 x - 6 \cos x) \\
 &= \sin x(4 \cos^2 x - 1 - 6 \cos x). \text{ Check other answers} \\
 & \text{with a grapher.}
 \end{aligned}$$

In #29–33, n represents any integer.

$$29. \quad \sin 2x = 0.5 \text{ when } 2x = \frac{\pi}{6} + 2n\pi \text{ or } 2x = \frac{5\pi}{6} + 2n\pi,$$

$$\text{so } x = \frac{\pi}{12} + n\pi \text{ or } x = \frac{5\pi}{12} + n\pi.$$

$$30. \quad \cos x = \frac{\sqrt{3}}{2} \text{ when } x = \pm \frac{\pi}{6} + 2n\pi$$

$$31. \quad \tan x = -1 \text{ when } x = -\frac{\pi}{4} + n\pi$$

$$32. \quad \text{If } \sin^{-1} x = \frac{\sqrt{2}}{2}, \text{ then } x = \sin \frac{\sqrt{2}}{2}.$$

$$33. \quad \text{If } \tan^{-1} x = 1, \text{ then } x = \tan 1.$$

$$\begin{aligned}
 34. \quad & 2 \cos 2x = 1 \\
 & \cos 2x = \frac{1}{2}
 \end{aligned}$$

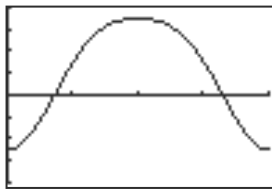
$$2 \cos^2 x - 1 = \frac{1}{2}$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

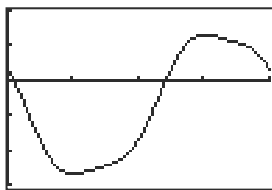
$$\text{So } x = \frac{\pi}{6} + 2n\pi \text{ or } x = \frac{5\pi}{6} + 2n\pi \text{ for } n \text{ any integer.}$$

35. $x \approx 1.12$ or $x \approx 5.16$



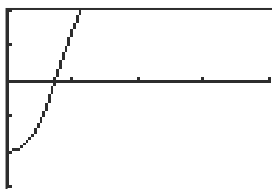
$[0, 2\pi]$ by $[-4, 4]$

36. $x \approx 0.14$ or $x \approx 3.79$



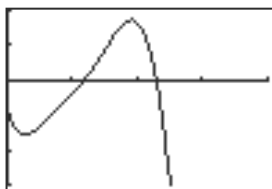
$[0, 2\pi]$ by $[-3, 2]$

37. $x \approx 1.15$



$[0, 2\pi]$ by $[-3, 2]$

38. $x \approx 1.85$ or $x \approx 3.59$



$[0, 2\pi]$ by $[-3, 2]$

39. $\cos x = \frac{1}{2}$, so $x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$.

40. $\sin 3x = (\sin x)(4 \cos^2 x - 1)$. This equation becomes $(\sin x)(4 \cos^2 x - 1) = \sin x$, or $2(\sin x)(2 \cos^2 x - 1) = 0$, so $\sin x = 0$ or

$$\cos x = \pm \frac{\sqrt{2}}{2}; x = 0, x = \frac{\pi}{4}, x = \frac{3\pi}{4}, x = \pi,$$

$$x = \frac{5\pi}{4}, \text{ or } x = \frac{7\pi}{4}.$$

41. The left side factors to $(\sin x - 3)(\sin x + 1) = 0$; only $\sin x = -1$ is possible, so $x = \frac{3\pi}{2}$.

42. $2 \cos^2 t - 1 = \cos t$, or $2 \cos^2 t - \cos t - 1 = 0$, or $(2 \cos t + 1)(\cos t - 1) = 0$. Then $\cos t = -\frac{1}{2}$

$$\text{or } \cos t = 1: t = 0, t = \frac{2\pi}{3} \text{ or } t = \frac{4\pi}{3}.$$

43. $\sin(\cos x) = 1$ only if $\cos x = \frac{\pi}{2} + 2n\pi$. No choice of n gives a value in $[-1, 1]$, so there are no solutions.

44. $\cos(2x) + 5 \cos x - 2 = 2 \cos^2 x - 1 + 5 \cos x - 2 = 2 \cos^2 x + 5 \cos x - 3 = 0$. $(2 \cos x - 1)(\cos x + 3) = 0$, so $\cos(x) = \frac{1}{2}$ and $\cos(x) = -3$. The latter is

extraneous so $x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$.

For #45–48, use graphs to suggest the intervals. To find the endpoints of the intervals, treat the inequalities as equations, and solve.

45. $\cos 2x = \frac{1}{2}$ has solutions $x = \frac{\pi}{6}, x = \frac{5\pi}{6}, x = \frac{7\pi}{6}$, and $x = \frac{11\pi}{6}$ in interval $[0, 2\pi)$. The solution set for the inequality is $0 \leq x < \frac{\pi}{6}$

$$\text{or } \frac{5\pi}{6} < x < \frac{7\pi}{6} \text{ or } \frac{11\pi}{6} < x < 2\pi;$$

$$\text{that is, } \left[0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right) \cup \left(\frac{11\pi}{6}, 2\pi\right).$$

46. $2 \sin x \cos x = 2 \cos x$ is equivalent to $(\cos x)(\sin x - 1) = 0$, so the solutions in $(0, 2\pi]$ are $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. The solution set for the inequality

$$\text{is } \frac{\pi}{2} < x < \frac{3\pi}{2}; \text{ that is, } \left(\frac{\pi}{2}, \frac{3\pi}{2}\right).$$

47. $\cos x = \frac{1}{2}$ has solutions $x = \frac{\pi}{3}$ and $x = \frac{5\pi}{3}$ in the interval $[0, 2\pi)$. The solution set for the inequality is $\frac{\pi}{3} < x < \frac{5\pi}{3}$; that is, $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$.

48. $\tan x = \sin x$ is equivalent to $(\sin x)(\cos x - 1) = 0$, so the only solution in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is $x = 0$. The solution set for the inequality is $-\frac{\pi}{2} < x < 0$; that is, $\left(-\frac{\pi}{2}, 0\right)$.

49. $y = 5 \sin(3x + \cos^{-1}(3/5)) \approx 5 \sin(3x + 0.93)$

50. $y = 13 \sin(2x - \cos^{-1}(5/13)) \approx 13 \sin(2x - 1.18)$

51. Given: $A = 79^\circ, B = 33^\circ, a = 7$ — an AAS case.

$$C = 180^\circ - (A + B) = 68^\circ;$$

$$b = \frac{a \sin B}{\sin A} = \frac{7 \sin 33^\circ}{\sin 79^\circ} \approx 3.9;$$

$$c = \frac{a \sin C}{\sin A} = \frac{7 \sin 68^\circ}{\sin 79^\circ} \approx 6.6.$$

52. Given: $a = 5, b = 8, B = 110^\circ$ — an SSA case. Using the law of sines: $h = a \sin B = 4.7; h < a < b$, so there is one triangle.

$$A = \sin^{-1}\left(\frac{a \sin B}{b}\right) \approx \sin^{-1}(0.587) \approx 36.0^\circ;$$

$$C = 180^\circ - (A + B) \approx 34.0^\circ;$$

$$c = \frac{b \sin C}{\sin B} \approx \frac{8 \sin 34.0^\circ}{\sin 110^\circ} \approx 4.8.$$

Using law of cosines: Solve the quadratic equation $8^2 = 5^2 + c^2 - 2(5)c \cos 110^\circ$, or $c^2 + (3.420)c - 39 = 0$; there is one positive solution:

$$c \approx 4.8. \text{ Since } \cos A = \frac{b^2 + c^2 - a^2}{2bc};$$

$$A \approx \cos^{-1}(0.809) \approx 36.0^\circ \text{ and } C = 180^\circ - (A + B) \approx 34.0^\circ.$$

53. Given: $a = 8, b = 3, B = 30^\circ$ — an SSA case. Using the law of sines: $h = a \sin B = 4; b < h$, so no triangle is formed.

Using the law of cosines: Solve the quadratic equation $3^2 = 8^2 + c^2 - 2(8)c \cos 30^\circ$, or $c^2 - (8\sqrt{3})c + 55 = 0$; there are no real solutions.

54. Given: $a = 14.7, A = 29.3^\circ, C = 33^\circ$ — an AAS case.

$$B = 180^\circ - (A + C) = 117.7^\circ, \text{ and}$$

$$b = \frac{a \sin B}{\sin A} = \frac{14.7 \sin 117.7^\circ}{\sin 29.3^\circ} \approx 26.6;$$

$$c = \frac{a \sin C}{\sin A} = \frac{14.7 \sin 33^\circ}{\sin 29.3^\circ} \approx 16.4.$$

55. Given: $A = 34^\circ, B = 74^\circ, c = 5$ — an ASA case.

$$C = 180^\circ - (A + B) = 72^\circ;$$

$$a = \frac{c \sin A}{\sin C} = \frac{5 \sin 34^\circ}{\sin 72^\circ} \approx 2.9;$$

$$b = \frac{c \sin B}{\sin C} = \frac{5 \sin 74^\circ}{\sin 72^\circ} \approx 5.1.$$

56. Given: $c = 41, A = 22.9^\circ, C = 55.1^\circ$ — an AAS case.

$$B = 180^\circ - (A + C) = 102^\circ;$$

$$a = \frac{c \sin A}{\sin C} = \frac{41 \sin 22.9^\circ}{\sin 55.1^\circ} \approx 19.5;$$

$$b = \frac{c \sin B}{\sin C} = \frac{41 \sin 102^\circ}{\sin 55.1^\circ} \approx 48.9.$$

57. Given: $a = 5, b = 7, c = 6$ — an SSS case.

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \approx \cos^{-1}(0.714) \approx 44.4^\circ;$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \approx \cos^{-1}(0.2) \approx 78.5^\circ;$$

$$C = 180^\circ - (A + B) \approx 57.1^\circ.$$

58. Given: $A = 85^\circ, a = 6, b = 4$ — an SSA case. Using the law of sines: $h = b \sin A \approx 4.0; h < b < a$, so there is one triangle.

$$B = \sin^{-1}\left(\frac{b \sin A}{a}\right) \approx \sin^{-1}(0.664) \approx 41.6^\circ;$$

$$C = 180^\circ - (A + B) \approx 53.4^\circ;$$

$$c = \frac{a \sin C}{\sin A} = \frac{6 \sin 53.4^\circ}{\sin 85^\circ} \approx 4.8.$$

Using the law of cosines: Solve the quadratic equation $6^2 = 4^2 + c^2 - 2(4)c \cos 85^\circ$, or $c^2 - (0.697)c - 20 = 0$; there is one positive solution:

$$c \approx 4.8. \text{ Since } \cos B = \frac{a^2 + c^2 - b^2}{2ac};$$

$$B \approx \cos^{-1}(0.747) \approx 41.6^\circ \text{ and } C = 180^\circ - (A + B) \approx 53.4^\circ.$$

59. $s = \frac{1}{2}(3 + 5 + 6) = 7;$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{7(7-3)(7-5)(7-6)}$$

$$= \sqrt{56} \approx 7.5.$$

60. $c \approx 7.672$ so area $\approx \sqrt{528.141} \approx 23.0$ (using Heron's formula). Or, use $A = \frac{1}{2}ab \sin C$.

61. $h = 12 \sin 28^\circ \approx 5.6$, so:

(a) $\approx 5.6 < b < 12$.

(b) $b \approx 5.6$ or $b \geq 12$.

(c) $b < 5.6$.

62. (a) $C = 180^\circ - (A + B) = 45^\circ$, so

$$AC = b = \frac{c \sin B}{\sin C} = \frac{80 \sin 65^\circ}{\sin 45^\circ} \approx 102.5 \text{ ft.}$$

(b) The distance across the canyon is $b \sin A \approx 96.4$ ft.

63. Given: $c = 1.75, A = 33^\circ, B = 37^\circ$ — an ASA case, so

$$C = 180^\circ - (A + B) = 110^\circ;$$

$$a = \frac{c \sin A}{\sin C} = \frac{1.75 \sin 33^\circ}{\sin 110^\circ} \approx 1.0;$$

$$b = \frac{c \sin B}{\sin C} = \frac{1.75 \sin 37^\circ}{\sin 110^\circ} \approx 1.1,$$

and finally, the height is $h = b \sin A = a \sin b \approx 0.6$ mi.

64. Given: $C = 70^\circ, a = 225, b = 900$ — an SAS case, so

$$AB = c = \sqrt{a^2 + b^2 - 2ab \cos C} \approx \sqrt{722,106.841} \approx 849.77 \text{ ft.}$$

65. Let $a = 8, b = 9$, and $c = 10$. The largest angle is opposite the largest side, so we call it C .

$$\text{Since } \cos C = \frac{a^2 + b^2 - c^2}{2ab}, C = \cos^{-1}\left(\frac{5}{16}\right)$$

$$\approx 71.790^\circ, 1.25 \text{ rad.}$$

66. The shorter diagonal splits the parallelogram into two (congruent) triangles with $a = 15, B = 40^\circ$,

and $c = 24$. The shorter diagonal has length $b = \sqrt{a^2 + c^2 - 2ac \cos B} \approx \sqrt{249.448} \approx 15.80$ ft.

Since adjacent angles are supplementary, the other angle is 140° . The longer diagonal splits the parallelogram into (two) congruent triangles with $a = 15, B = 140^\circ$, and $c = 24$, so the longer diagonal length is

$$b = \sqrt{a^2 + c^2 - 2ac \cos B} \approx \sqrt{1352.552} \approx 36.78 \text{ ft.}$$

67. (a) The point (x, y) has coordinates $(\cos \theta, \sin \theta)$, so the

bottom is $b_1 = 2$ units wide, the top is

$b_2 = 2x = 2 \cos \theta$ units wide, and the height is

$h = y = \sin \theta$ units. Either use the formula for the

area of a trapezoid, $A = \frac{1}{2}(b_1 + b_2)h$, or notice that

the trapezoid can be split into two triangles and a

rectangle. Either way:

$$A(\theta) = \sin \theta + \sin \theta \cos \theta = \sin \theta(1 + \cos \theta)$$

$$= \sin \theta + \frac{1}{2} \sin 2\theta.$$

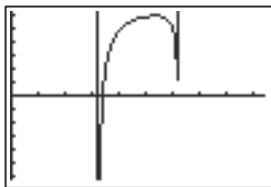
- (b) The maximizing angle is $\theta = \frac{\pi}{3} = 60^\circ$; the maximum

area is $\frac{3}{4}\sqrt{3} \approx 1.30$ square units.

68. (a) Substituting the values of a and b :

$$S(\theta) = 6.825 + 0.63375(-\cot \theta + \sqrt{3} \cdot \csc \theta)$$

$$= 6.825 + \frac{0.63375(\sqrt{3} - \cos \theta)}{\sin \theta}$$



$[0, 9.4]$ by $[-6.2, 6.2]$

- (b) Considering only angles between 0 and π , the minimum occurs when $\theta \approx 0.955$ rad $\approx 54.75^\circ$.
 (c) The minimum value of S is approximately $S(0.955) \approx 7.72$ in.²

69. (a) Split the quadrilateral in half to leave two (identical) right triangles, with one leg 4000 mi, hypotenuse $4000 + h$ mi, and one acute angle $\theta/2$.

$$\text{Then } \cos \frac{\theta}{2} = \frac{4000}{4000 + h}; \text{ solve for } h \text{ to leave}$$

$$h = \frac{4000}{\cos(\theta/2)} - 4000 = 4000 \sec \frac{\theta}{2} - 4000$$

(b) $\cos \frac{\theta}{2} = \frac{4000}{4200}$, so $u = 2 \cos^{-1}\left(\frac{20}{21}\right) \approx 0.62 \approx 35.51^\circ$.

70. Rewrite the left side of the equation as follows:

$$\begin{aligned} & \sin x - \sin 2x + \sin 3x \\ &= \sin x - 2 \sin x \cos x + \sin(2x + x) \\ &= \sin x - 2 \sin x \cos x + \sin 2x \cos x + \cos 2x \sin x \\ &= \sin x - 2 \sin x \cos x + 2 \sin x \cos^2 x + (\cos^2 x - \sin^2 x) \sin x \\ &= \sin x - 2 \sin x \cos x + 2 \sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x \\ &= \sin x - 2 \sin x \cos x + 3 \sin x \cos^2 x - \sin^3 x \\ &= \sin x (1 - 2 \cos x + 3 \cos^2 x - \sin^2 x) \\ &= \sin x ((1 - \sin^2 x) - 2 \cos x + 3 \cos^2 x) \\ &= \sin x (\cos^2 x - 2 \cos x + 3 \cos^2 x) \\ &= \sin x (4 \cos^2 x - 2 \cos x) \\ &= 2 \sin x \cos x (2 \cos x - 1) \\ &= \sin 2x (2 \cos x - 1) \end{aligned}$$

So $\sin 2x = 0$ or $2 \cos x - 1 = 0$. So $x = n\frac{\pi}{2}$ or

$$x = \pm \frac{\pi}{3} + 2n\pi, n \text{ an integer.}$$

71. The hexagon is made up of 6 equilateral triangles; using Heron's formula (or some other method), we find that each triangle has area $\sqrt{24(24 - 16)^3}$
 $= \sqrt{12,288} = 64\sqrt{3}$. The hexagon's area is therefore $384\sqrt{3}$ cm², and the radius of the circle is 16 cm, so the area of the circle is 256π cm², and the area outside the hexagon is $256\pi - 384\sqrt{3} \approx 139.14$ cm².

72. The pentagon is made up of 5 triangles with base length 12 cm and height of $6 \tan 54^\circ$ cm, so its area is $5 \cdot \frac{1}{2} \cdot 12 \cdot 6 \tan 54^\circ = 180 \tan 54^\circ$ cm². The radius of the circle is the height of those triangles, so its area is $\pi(6 \tan 54^\circ)^2$. The desired area is area of pentagon - area of circle = $180 \tan 54^\circ - \pi(6 \tan 54^\circ)^2 \approx 33.49$ cm².

73. The volume of a cylinder with radius r and height h is $V = \pi r^2 h$, so the wheel of cheese has volume $\pi(9^2)(5) = 405\pi$ cm³; a 15° wedge would have fraction $\frac{15}{360} = \frac{1}{24}$ of that volume, or $\frac{405\pi}{24} \approx 53.01$ cm³.

74. (a) $\frac{1}{2}(\cos(u - v) - \cos(u + v))$

$$= \frac{1}{2}(\cos u \cos v + \sin u \sin v - (\cos u \cos v - \sin u \sin v))$$

$$= \frac{1}{2}(2 \sin u \sin v)$$

$$= \sin u \sin v$$

(b) $\frac{1}{2}(\cos(u - v) + \cos(u + v))$

$$= \frac{1}{2}(\cos u \cos v + \sin u \sin v + \cos u \cos v - \sin u \sin v)$$

$$= \frac{1}{2}(2 \cos u \cos v)$$

$$= \cos u \cos v$$

(c) $\frac{1}{2}(\sin(u + v) + \sin(u - v))$

$$= \frac{1}{2}(\sin u \cos v + \cos u \sin v + \sin u \cos v - \cos u \sin v)$$

$$= \frac{1}{2}(2 \sin u \cos v)$$

$$= \sin u \cos v$$

75. (a) By the product-to-sum formula in Exercise 74 (c),

$$\begin{aligned} & 2 \sin \frac{u+v}{2} \cos \frac{u-v}{2} \\ &= 2 \cdot \frac{1}{2} \left(\sin \frac{u+v+u-v}{2} + \sin \frac{u+v-(u-v)}{2} \right) \end{aligned}$$

$$= \sin u + \sin v$$

- (b) By the product-to-sum formula in Exercise 74 (c),

$$\begin{aligned} & 2 \sin \frac{u-v}{2} \cos \frac{u+v}{2} \\ &= 2 \cdot \frac{1}{2} \left(\sin \frac{u-v+u+v}{2} + \sin \frac{u-v-(u+v)}{2} \right) \end{aligned}$$

$$= \sin u + \sin(-v)$$

$$= \sin u - \sin v$$

- (c) By the product-to-sum formula in Exercise 74 (b),

$$\begin{aligned} & 2 \cos \frac{u+v}{2} \cos \frac{u-v}{2} \\ &= 2 \cdot \frac{1}{2} \left(\cos \frac{u+v-(u-v)}{2} + \cos \frac{u+v+u-v}{2} \right) \end{aligned}$$

$$= \cos v + \cos u$$

$$= \cos u + \cos v$$

- (d) By the product-to-sum formula in 74 (a),

$$-2 \sin \frac{u+v}{2} \sin \frac{u-v}{2}$$

$$\begin{aligned}
 &= -2 \cdot \frac{1}{2} \left(\cos \frac{u+v-(u-v)}{2} \right. \\
 &\quad \left. - \cos \frac{u+v+u-v}{2} \right) \\
 &= -(\cos v - \cos u) \\
 &= \cos u - \cos v
 \end{aligned}$$

76. Pat faked the data. The law of cosines can be solved to

show that $x = 12\sqrt{\frac{2}{1 - \cos \theta}}$. Only Carmen's values are consistent with the formula.

77. (a) Any inscribed angle that intercepts an arc of 180° is a right angle.

(b) Two inscribed angles that intercept the same arc are congruent.

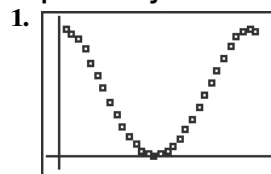
(c) In right $\triangle A'BC$, $\sin A' = \frac{\text{opp}}{\text{hyp}} = \frac{a}{d}$.

(d) Because $\angle A'$ and $\angle A$ are congruent,

$$\frac{\sin A}{a} = \frac{\sin A'}{a} = \frac{a/d}{a} = \frac{1}{d}.$$

(e) Of course. They both equal $\frac{\sin A}{a}$ by the law of sines.

Chapter 5 Project



$[-2, 34]$ by $[-0.1, 1.1]$

2. We set the amplitude as half the difference between the maximum value, 1.00, and the minimum value, 0.00, so $a = 0.5$. And we set the average value as the average of the maximum and minimum values, so $k = 0.5$. Since $\cos(b(x - h))$ has a period of about 30, we set $b = 2\pi/30.5 = 0.2$. Experimenting with the graph suggests that h should be about -0.5 . So the equation is $y = 0.5 \cos(0.2(x + 0.05)) + 0.5$.
5. We set the amplitude as half the difference between the maximum value, 1.00, and the minimum value, 0.00, so $a = 0.5$. And we set the average value as the average of the maximum and minimum values, so $k = 0.5$. Since $\sin(b(x - h))$ has a period of about 30, we set $b = 2\pi/30 = 0.2$. Experimenting with the graph suggests that h should be about -8.4 . So the equation is $y = 0.5 \sin(0.2(x + 8.4)) + 0.5$.
7. The graphs are the same.