5.5 Day 2 and 3

Sunday, November 20, 2016 4:30 PM

5.5b-c Differentials and Estimating Change

DEFINITION Differentials

Let $y = f(x)$ be a differentiable function. The **differential dx** is an independent variable. The **differential dy** is

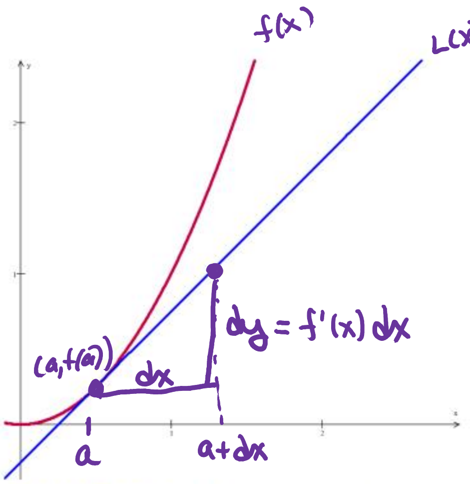
$$dy = f'(x)\, dx.$$

$$\frac{dy}{dx} = \frac{df}{dx} = f'(x)$$

$$\therefore \; dy = f'(x)\, dx$$

or

$$df = f'(x)\, dx$$



EXAMPLE 6 Finding the Differential *dy*

Find the differential dy and evaluate dy for the given values of x and dx.

(a) $y = x^5 + 37x$, $x = 1$, $dx = 0.01$ (b) $y = \sin 3x$, $x = \pi$, $dx = -0.02$

(c) $x + y = xy$, $x = 2$, $dx = 0.05$

a. $\frac{dy}{dx} = 5x^4 + 37$

$dy = (5x^4 + 37)dx$

$dy = (5(1)^4 + 37).01 \rightarrow dy = .42$

b. $\frac{dy}{dx} = 3\cos 3x$

☆ $dy = 3\cos 3x\, dx$

$dy = 3\cos 3(\pi)(-.02)$

$= -3(-.02) = .06$

$dy = .06$

c. $1 + \frac{dy}{dx} = y(1) + x\frac{dy}{dx}$ ☆

$-x\frac{dy}{dx} + \frac{dy}{dx} = y - 1$

$\frac{dy}{dx}(-x+1) = y - 1$

$\frac{dy}{dx} = \frac{y-1}{-x+1}$

$\rightarrow dy = \left(\frac{y-1}{-x+1}\right)dx$

$dy = \frac{2-1}{-2+1}(.05)$

$= \frac{1}{-1}(.05)$ $\boxed{dy = -.05}$

$2 + y = 2y$

$2 = y$

Notation: $df = f'(x)\,dx$

let $f = 3x^2 - 6$

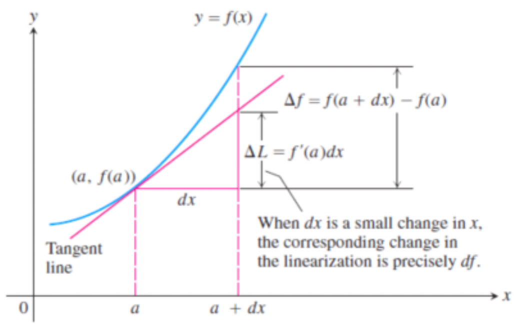
$df = d(3x^2 - 6) = 6x\,dx$

EXAMPLE 7 Finding Differentials of Functions

(a) $d(\tan 2x) = \sec^2(2x)\,d(2x) = 2\sec^2 2x\,dx$

(b) $d\left(\dfrac{x}{x+1}\right) = \dfrac{(x+1)\,dx - x\,d(x+1)}{(x+1)^2} = \dfrac{x\,dx + dx - x\,dx}{(x+1)^2} = \dfrac{dx}{(x+1)^2}$

Estimating Changes in function values using Differentials.



EXAMPLE 8 **Estimating Change With Differentials**

The radius r of a circle increases from $a = 10$ m to 10.1 m (Figure 4.54). Use dA to estimate the increase in the circle's area A. Compare this estimate with the true change ΔA, and find the approximation error.

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

Estimated Change

$dA = 2\pi r\,dr$ $\quad r = 10 \quad dr = .1$

$dA = 2\pi(10)(.1)$

$dA = 2\pi\ m^2$

Approximation Error:

$.01\pi\ m^2 \approx .0314\ m^2$

True change:

$A(10.1) - A(10) = \Delta A$

$10.1^2\pi - 10^2\pi$

$102.01\pi - 100\pi = 2.01\pi = \Delta A$

Absolute, Relative, and Percentage Change

As we move from a to a nearby point $a + dx$, we can describe the change in f in three ways:

	True	Estimated
Absolute change	$\Delta f = f(a + dx) - f(a)$	$df = f'(a)\,dx$
Relative change	$\frac{\Delta f}{f(a)}$	$\frac{df}{f(a)}$
Percentage change	$\frac{\Delta f}{f(a)} \times 100$	$\frac{df}{f(a)} \times 100$

EXAMPLE 9 **Changing Tires**

Inflating a bicycle tire changes its radius from 12 inches to 13 inches. Use differentials to estimate the absolute change, the relative change, and the percentage change in the perimeter of the tire.

$P = 2\pi r$

$\frac{dP}{dr} = 2\pi$

Estimated change

$dP = 2\pi dr$ $\quad r = 12 \quad dr = 1$

$dP = 2\pi(1)$

$dP = 2\pi$ inches

True change:

$P(13) - P(12) = 26\pi - 24\pi = 2\pi$ inches

% change

$\frac{\Delta P}{P(12)} = \frac{2\pi}{24\pi} = \frac{1}{12} \approx .08\overline{3}$ or 8.3%

EXAMPLE 10 **Estimating the Earth's Surface Area**

Suppose the earth were a perfect sphere and we determined its radius to be 3959 ± 0.1 miles. What effect would the tolerance of ± 0.1 mi have on our estimate of the earth's surface area?

$S = 4\pi r^2$

$\frac{dS}{dr} = 8\pi r$

$dS = 8\pi r\, dr$

$dS = 8\pi(3959)(.1)$

$dS \approx 9950 \text{ mi}^2$

An error of 0.1 in the measured radius results in an estimated error of 9950 mi² in the earth's surface area.

EXAMPLE 11 Determining Tolerance

About how accurately should we measure the radius r of a sphere to calculate the surface area $S = 4\pi r^2$ within 1% of its true value?