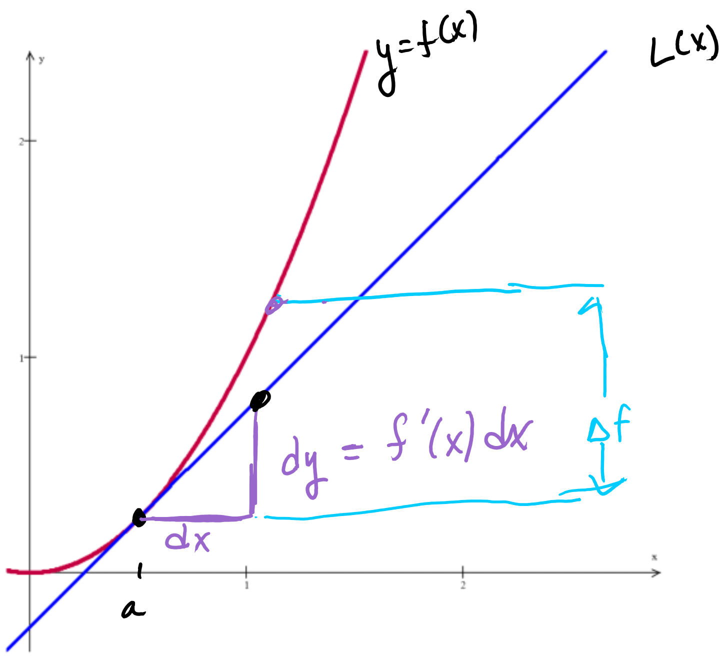
5.5 Day 2 (thurs 10/17)

Wednesday, October 16, 2019 12:51 PM

DEFINITION Differentials

Let $y = f(x)$ be a differentiable function. The **differential dx** is an independent variable. The **differential dy** is

$$dy = f'(x)\,dx.$$



slope of $L(x) = \frac{dy}{dx}$

$\frac{dy}{dx} = f'(x)$

$\therefore\ dy = f'(x)\,dx$

EXAMPLE 5 Finding the Differential dy

Find the differential dy and evaluate dy for the given values of x and dx.

(a) $y = x^5 + 37x$, $x = 1$, $dx = 0.01$ **(b)** $y = \sin 3x$, $x = \pi$, $dx = -0.02$

(c) $x + y = xy$, $x = 2$, $dx = 0.05$

a. $\frac{dy}{dx} = 5x^4 + 37$

$dy = (5x^4 + 37) \cdot dx$

$= (5 + 37)(.01) = .42$

b. $\frac{dy}{dx} = 3\cos(3x)$

$dy = 3\cos(3x)\,dx$

$dy = 3\cos(3\pi)(-0.02)$

$= -3(-0.02) = .06$

c. $1 + y' = xy' + y$

$y' - xy' = y - 1$

$y'(1-x) = y - 1$

$y' = \dfrac{y-1}{1-x}$

$dy = \left(\dfrac{y-1}{1-x}\right) dx$

$x + y = xy$

$2 + y = 2y$

$2 = y$

$dy = \dfrac{2-1}{1-2}(.05)$

$dy = -.05$

Notation: $\frac{df}{dx} = f'(x) \Rightarrow \underbrace{df = f'(x)\,dx}_{\text{differential}}$

EXAMPLE 6 Finding Differentials of Functions

(a) $d(\tan 2x) = \sec^2(2x)\,d(2x) = 2\sec^2 2x\,dx$

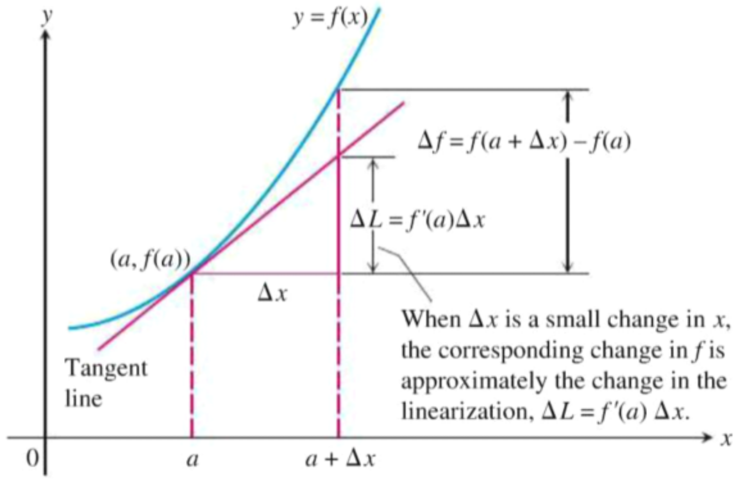
$= f'(x) \cdot dx$

(b) $d\left(\frac{x}{x+1}\right) = \frac{(x+1)\,dx - x\,d(x+1)}{(x+1)^2} = \frac{x\,dx + dx - x\,dx}{(x+1)^2} = \frac{dx}{(x+1)^2}$

$d\left(\frac{x}{x+1}\right)$ = find the differential

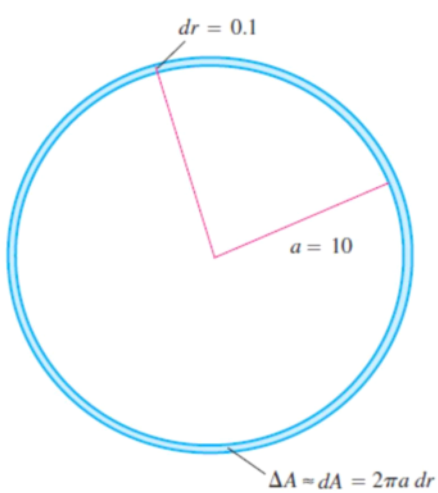
$dy = \frac{1}{(x+1)^2}\,dx$

Estimating Changes in function values using Differentials.



EXAMPLE 7 Estimating Change

The radius r of a circle increases from $a = 10$ m to 10.1 m (Figure 5.47). Use the linearization to estimate the increase in the circle's area A. Compare this estimate with the true change ΔA, and find the approximation error.



$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$dA = 2\pi r\, dr$

Estimated Change

$dA = 2\pi(10)(.1)$

$= 2\pi \text{ m}^2$

True change

$A(10) = 100\pi \text{ m}^2$

$A(10.1) = 102.01\pi \text{ m}^2$

$\doteq 2.01\pi \text{ m}^2$

Absolute, Relative, and Percentage Change

As we move from a to a nearby point $a + \Delta x$, we can describe the change in f in three ways:

	True	Estimated
Absolute change	$\Delta f = f(a + \Delta x) - f(a)$	$\Delta f \approx f'(a)\Delta x$
Relative change	$\dfrac{\Delta f}{f(a)}$	$\dfrac{f'(a)}{f(a)}\Delta x$
Percentage change	$\dfrac{\Delta f}{f(a)} \times 100$	$\dfrac{f'(a)}{f(a)}\Delta x \times 100$

EXAMPLE 8 Changing Tires

Inflating a bicycle tire changes its radius from 12 inches to 13 inches. Use the linearization to estimate the absolute change, the relative change, and the percentage change in the perimeter of the tire.

EXAMPLE 11 Unclogging Arteries

In the late 1830s, the French physiologist Jean Poiseuille ("pwa-ZOY") discovered the formula we use today to predict how much the radius of a partially clogged artery has to be expanded to restore normal flow. His formula,

$$V = kr^4,$$

says that the volume V of fluid flowing through a small pipe or tube in a unit of time at a fixed pressure is a constant times the fourth power of the tube's radius r. How will a 10% increase in r affect V?

EXAMPLE 12 Finding Depth of a Well

You want to calculate the depth of a well from the equation $s = 16t^2$ by timing how long it takes a heavy stone you drop to splash into the water below. How sensitive will your calculations be to a 0.1-sec error in measuring the time?