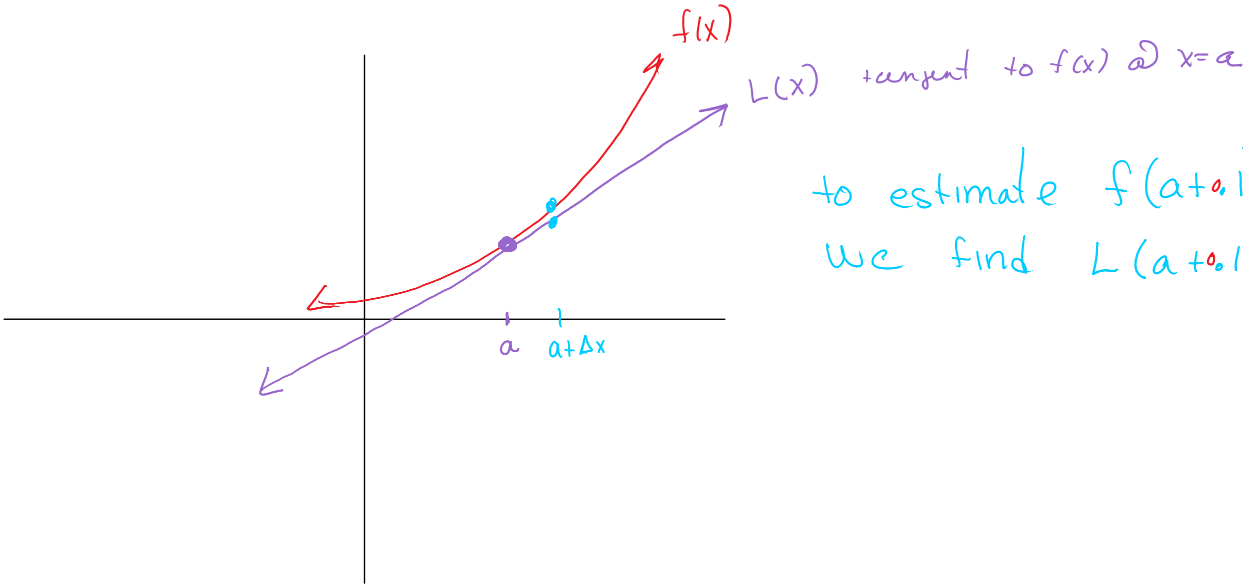
5.5 Day 1 Linearization & Differentials

Wednesday, October 16, 2019 8:15 AM



to estimate $f(a+0.1)$

we find $L(a+0.1)$

Def: If f is differentiable @ $x=a$, then the equation of the tangent line

$$y - f(a) = f'(a)(x-a)$$

$$L(x) = f'(a)(x-a) + f(a)$$

defines the linearization of f @ $x=a$.

This can be used to approximate value close to $x=a$ on the curve.

☆ linear approximation

Ex: Find the linearization of $f(x)=\sqrt{x}$ @ $x=4$ and then approx. $\sqrt{4.01}$ w/o a calc.

1. $f'(x) = \frac{1}{2}x^{-1/2}$

2. $m = f'(4) = \frac{1}{2}(4)^{-1/2} = 1/4$

3. p.o.t $f(4) = \sqrt{4} = 2$ $(4, 2)$

4. $L(x) = \frac{1}{4}(x-4) + 2$

5. approx. $L(4.01) = 0.25(4.01-4) + 2$

$= 0.25(.01) + 2$

$= 2.0025$

$\sqrt{4.01} \approx 2.0025$

Error: $|\text{true value} - \text{approx}|$

$|\sqrt{4.01} - 2.0025|$ Less than 10^{-5} $\quad 1.56 \times 10^{-6}$

★ when is the linear approx. an under/over estimate?

- Concavity
  - up → under
  - down → over

Ex: $f(x) = x^3 + 2x + 3$

a. Find the linearization @ $x = 2$

b. approx $f(2.01)$ using a linear approx.

c. Determine if the approx. is an under/over estimate. Justify

a. $f'(x) = 3x^2 + 2$ $\quad f'(2) = 14$ $\quad f(2) = 15$

$$\boxed{L(x) = 14(x-2) + 15}$$

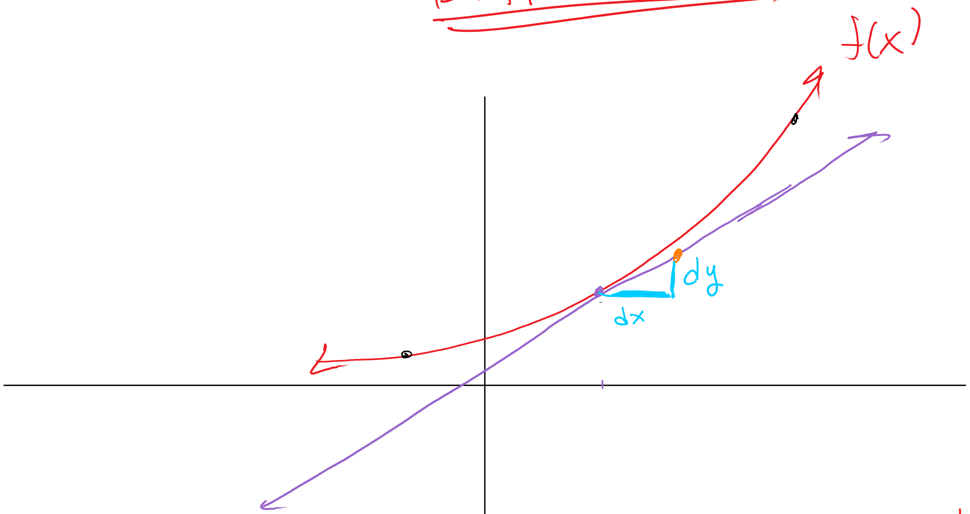
b. $L(2.01) = 14(.01) + 15 = 15.14$

c. $f''(x) = 6x$ $\quad f''(2.01) > 0$ $\quad f''$: N | P at 0

$L(2.01) \approx 15.14$ is an under estimate $f(2.01)$ b/c f is ccu $(0, \infty)$ b/c $f'' > 0$ over the interval.

d. Error $|f(2.01) - L(2.01)|$ less than 10^{-3}

Differentials



$dy \Rightarrow$ change in y
$dx \Rightarrow$ change in x

Derivative $\rightarrow \frac{dy}{dx}$ tells you slope

Differentials $\rightarrow dx$ & dy, which are separate components of dy/dx and they tell us about change.

Ex: $y = 4x^2 + 5$
suppose $x = 3$ & $dx = 0.01$, find dy

1. $\frac{dy}{dx} = 8x$ (Derivative)

2. Differential $dy = 8x \cdot dx$

$$dy = 8(3)(.01) = 24(.01) = .24$$

when x increases by $.01 \rightarrow y$ changes by approx. 0.24

@ $x = 3$