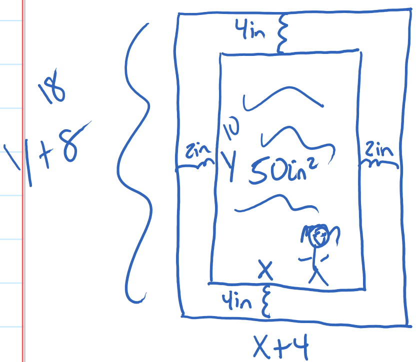
5.4B Notes

Friday, October 13, 2017 6:55 AM

Section 5.4 - More Optimization Problems!!!

1. You are designing a rectangular poster to contain 50 square inches of printing with a 4-in margin at the top and bottom and a 2 in margin on each side. What overall dimensions will minimize the amount of paper used? Minimize area!



$A = (x+4)(y+8)$

$A(x) = (x+4)\left(\frac{50}{x} + 8\right)$

$A(x) = 50 + 8x + \frac{200}{x} + 32$

$A(x) = 82 + 8x + \frac{200}{x}$

$A'(x) = 8 - \frac{200}{x^2} = 0$

$8 = \frac{200}{x^2}$

$8x^2 = 200$

$\sqrt{x^2} = \sqrt{25}$

C.P.: $x = 5$

$50 = x \cdot y$ $\quad y = \frac{50}{5} = 10$

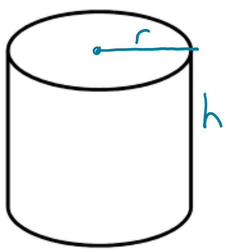
$y = \frac{50}{x} > 0$

A': 0, 5; test values 1 ($-$), 10 ($+$)

Dim: 9in x 18in

$\boxed{\text{The area is min. when } x = 5\text{in since } A' \text{ changed from } - \text{ to } + \text{ @ } x = 5 \text{ and } x = 5 \text{ is the only c.p. on } (0,\infty).}$

2. You have been asked to design a 1000 cubic centimeter cylindrical can. What dimensions of the can will require the least amount of material to make it? (Hint: What are we going to minimize?) Minimize S.A.



$A = 2\pi r^2 + 2\pi r h$

$A(r) = 2\pi r^2 + 2\cancel{\pi r} \cdot \frac{1000}{\cancel{\pi} r^{\cancel{2}}}$

$A(r) = 2\pi r^2 + \frac{2000}{r}$

$A'(r) = 4\pi r - \frac{2000}{r^2} = 0$

$4\pi r = \frac{2000}{r^2}$

$4\pi r^3 = 2000$

$r^3 = 159.154$

C.P.: $r \approx 5.419$

$1000 = \pi r^2 h$

$h = \frac{1000}{\pi r^2} > 0$

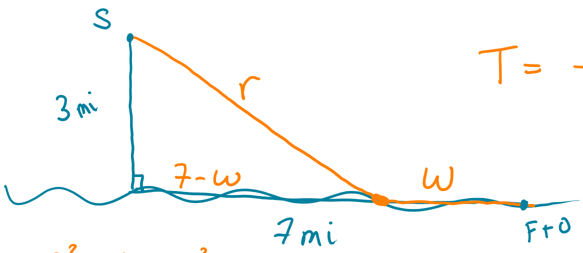
A': 0, 5.419; test values 1 ($-$), 10 ($+$)

$\boxed{\begin{array}{l} r \approx 5.419\text{cm} \\ h \approx 10.839\text{cm} \end{array}}$

3. Mrs. Saller is in a boat 3 miles offshore and wants to meet up with Mr. Frees and Ms. Orloff in a village 7 miles down a straight shoreline from the point nearest the boat. Mrs. Saller is really strong, but walks very slow, so she can row 3 mph and can walk 4 mph. Where should she land the boat to reach her two colleagues in the least amount of time? Minimize time!

$D = r \cdot t$

$t = \frac{D}{r}$



$3^2 + (7-w)^2 = r^2$

$r = \sqrt{9 + (7-w)^2}$

$T = \frac{r}{3} + \frac{w}{4}$

$T(w) = \frac{\sqrt{9+(7-w)^2}}{3} + \frac{w}{4}$ $\quad (9+(7-w)^2)^{\frac{1}{2}}$

$T'(w) = \frac{1}{4} + \frac{\cancel{\frac{1}{2}}(9+(7-w)^2)^{-1/2} \cdot (\cancel{2}(7-w) \cdot -1)}{3}$

$= \frac{1}{4} - \underbrace{\frac{(7-w)(9+(7-w)^2)^{-1/2}}{3}}_{Y_1} = \underbrace{0}_{Y_2}$

$-$ $+$ A'

0 3.598 7

C.P.: $w \approx 3.598$

$\boxed{\approx 3.402 \text{ mi downshore}}$

profit = revenue − cost

$p(x) = r(x) - c(x) \Rightarrow p'(x) = r'(x) - c'(x) = 0$

Maximum profit occurs when $r'(x) = c'(x)$ (check correct sign change)

$r'(x) = c'(x)$

marginal revenue   marginal cost

Minimizing average cost occurs when

4. Suppose that Saller Inc. has a revenue of $r(x) = 9x$ and has a cost of $c(x) = x^3 - 6x^2 + 15x$ where x represents thousands of units of math facts (yes, there are that many). Is there a production level that maximizes profit? If so, what is it? Is there a production level that minimizes average cost? If so, what is it?

$$p(x) = 9x - (x^3 - 6x^2 + 15x)$$

$$p(x) = -x^3 + 6x^2 - 6x$$

$$p'(x) = \underbrace{-3x^2 + 12x - 6}_{Y_1} = \underbrace{0}_{Y_2}$$

C.P.: $x \approx 3.414,\ 0.586$

p': $-$ on $(0, 0.586)$, $+$ on $(0.586, 3.414)$, $-$ after 3.414

$$\boxed{x \approx 3.414 \text{ thousands of math facts}}$$

The max profit occurs when $x \approx 3.414$ since p' changed from $+$ to $-$ at $x \approx 3.414$ and $p' < 0$ to the right of $x \approx 3.414$ for all values of $x > 3.414$ and the other C.P is a minimum since p' changed from $-$ to $+$ @ that pt.