5.4A Notes

AP Calculus BC Section 5.4 Day 1 – Optimization! → Finding the abs. max or min amount of a real-world situation to optimize that amount.

Set up an equation in terms of one variable for each of the following problems. Then find the maximum for each problem and justify your answers!

1.

RIVER

x x

y
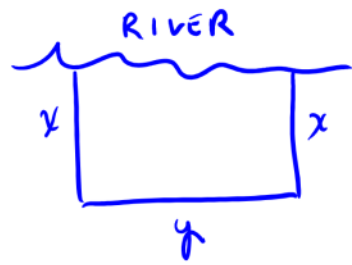
A wire fence to keep horses contained is to border a river. No fencing is necessary along the river. If 600 m of fencing is available, write an equation for the area of the rectangle. Maximize the area. Justify!

$A = x \cdot y$

$A(x) = x(600 - 2x)$

$A(x) = 600x - 2x^2$

$A'(x) = 600 - 4x = 0$

$600 = 4x$

C.P.: $x = 150$

$A(150) = 150(600 - 300)$

$= 150 \cdot 300 = 45,000 \, m^2$

$2x + y = 600$

$y = 600 - 2x > 0$

$600 > 2x \quad 300 > x$

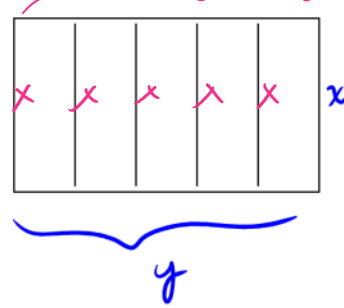$\xleftarrow{\quad} \underset{0}{\quad} + \underset{150}{\quad} \underset{200}{\overline{\quad}} \underset{300}{\quad} \to A'$
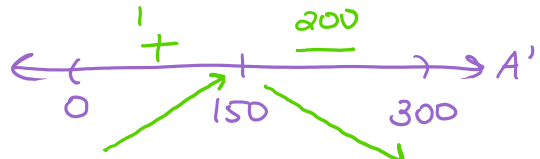
The area is max. of 45,000 m² when x = 150 m since A' changed from + to − at x = 150 and x = 150 is the only C.P on (0, 300).

2. A small motel is to be built as shown in the sketch with two long walls y feet each and 6 short walls x feet long each. The total length of the walls is to be 300 feet. Write an equation for the square feet of area taken up by the motel. Maximize the area! Justify!

x x x x x x

y

$A = x \cdot y$

$A(x) = x(150 - 3x)$

$A(x) = 150x - 3x^2$

$A'(x) = 150 - 6x = 0$

$150 = 6x$

C.P.: $x = 25$

$A(25) = 25(150 - 75)$

$= 25 \cdot 75$

$= 1,875 \, ft^2$

$6x + 2y = 300$

$3x + y = 150$

$y = 150 - 3x > 0$

$150 > 3x$

$\xleftarrow{\quad} \underset{0}{\quad} + \underset{25}{\quad} \underset{30}{\overline{\quad}} \underset{50}{\quad} \to A'$
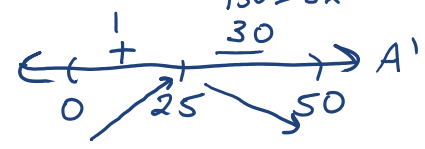
The area is max of 1875 ft² when x = 25 ft since A' changed from + to − @ x = 25 and x = 25 is the only C.P. on (0, 50).

3. Two **positive** numbers add to 30, write an equation for the product of the two numbers. $\mathcal{M}$aximize product! Justify!

#'s → a and b

$a + b = 30$
$a = 30 - b$

$P = a \cdot b$
$P(b) = b(30 - b)$
$P(b) = 30b - b^2$
$P'(b) = 30 - 2b = 0$
$\qquad 30 = 2b$
c.p: $b = 15$
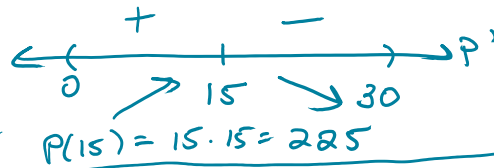
$P(15) = 15 \cdot 15 = 225$
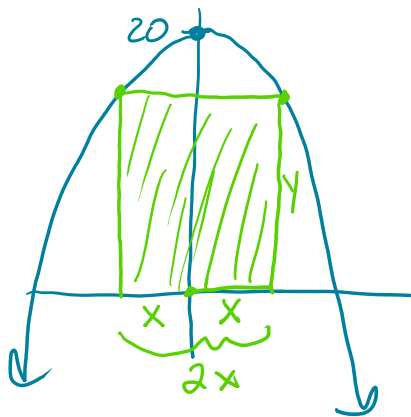
The product has a max of 225 when $b = 15$ since P' changed from + to − @ $b = 15$ and $b = 15$ is the only C.P. on $(0, 30.$

4. A rectangle has its base on the x-axis and its upper vertices on the parabola on the parabola $y = -x^2 + 20$. Write an equation for the area of the rectangle.
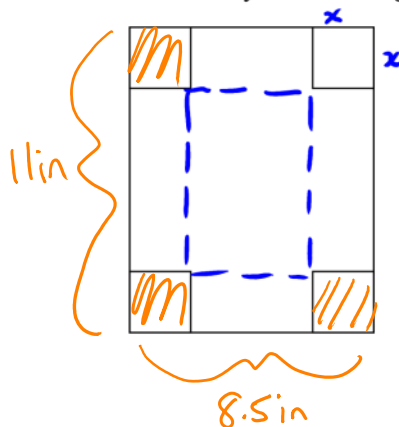

$A = 2x \cdot y$
$A(x) = 2x(-x^2 + 20)$
$A(x) = -2x^3 + 40x$

5. Given a sheet of paper, 8.5 inches by 11 inches. Create a box by cutting from each corner a square of dimension x by x and folding up along the dotted lines. Write an equation for the volume of the box.

11in

8.5 in

$V = \ell \cdot w \cdot h$
$V(x) = x \cdot (11 - 2x)(8.5 - 2x)$
$V(x) = (11x - 2x^2)(8.5 - 2x)$
$V(x) = -22x^2 + 93.5x - 17x^2 + 4x^3$
$V(x) = 4x^3 - 39x^2 + 93.5x$