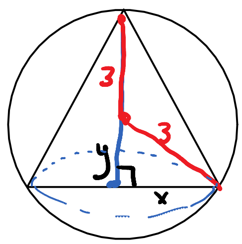
4.4 day 5 p. 229: 36, 40, 41, 47

Saturday, November 24, 2012
2:08 PM

36.



Volume of a cone
$V = \frac{1}{3}\pi r^2 h$

$x^2 + y^2 = 9$ $\boxed{x = \sqrt{9 - y^2}}$

x = radius of the cone

$h = 3 + y$

$V = \frac{1}{3}\pi x^2 (3+y)$

$= \frac{1}{3}\pi \left(\sqrt{9-y^2}\right)^2 (3+y)$

$V = \frac{1}{3}\pi (9-y^2)(3+y) = \frac{1}{3}\pi (27 + 9y - 3y^2 - y^3)$ for $-3 < y < 3$

$V' = \frac{1}{3}\pi \frac{d}{dy}(27 + 9y - 3y^2 - y^3)$

$= \frac{1}{3}\pi (9 - 6y - 3y^2)$

$0 = 9 - 6y - 3y^2)$ $0 = -3(y^2 + 2y - 3)$

$= -3(y+3)(y-1)$

c.v

$\cancel{y = -3}$ $y = 1$

V': -3 (P 1 N) 3

☆ $V(1) = \frac{32\pi}{3}$ maximum volume

The cone has a maximum volume of $\frac{32\pi}{3}$ when $y = 1$ because V' goes from + to − @ $y = 1$.

40. $i = 2\cos t + 2\sin t$

$i' = -2\sin t + 2\cos t$

$0 = -2(\sin t - \cos t)$

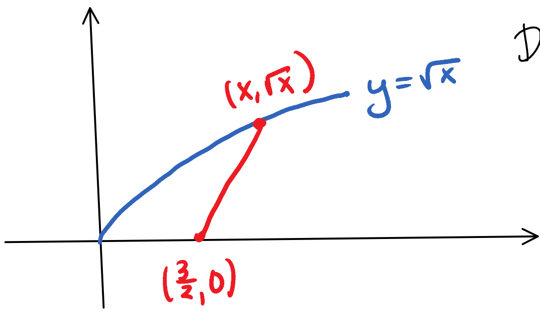
$0 = \sin t - \cos t$ or $\cos t = \sin t$ $t = \frac{\pi}{4} + \pi k$ where k is an integer

largest magnitude $i = \left|2\cos\frac{\pi}{4} + 2\sin\frac{\pi}{4}\right|$

$= 2 \cdot \frac{1}{\sqrt{2}} + 2 \cdot \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2}$ or $2\sqrt{2}$ amps ☆

$$= 2\cdot\frac{1}{\sqrt{2}} + 2\cdot\frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} \text{ or } \boxed{2\sqrt{2} \text{ amps}}$$

41.



D(x) = The square of the distance

$$D(x) = \left(x - \tfrac{3}{2}\right)^2 + \left(\sqrt{x} - 0\right)^2$$

$$= x^2 - 3x + \frac{9}{4} + x$$

$$= x^2 - 2x + 9/4$$

$$D'(x) = 2x - 2 \qquad 0 = 2x - 2$$

$$x = 1$$

D′ ——N——|1——P——

because D(x) represents the square of the distance

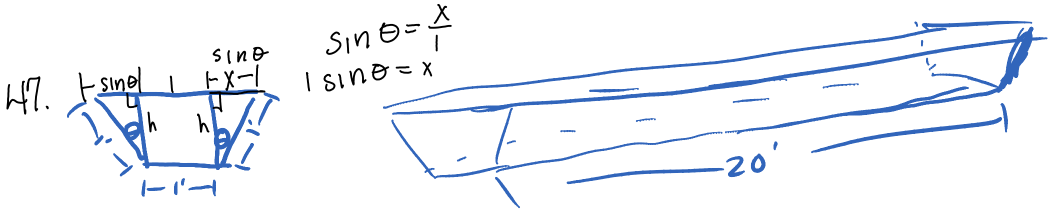
minimum distance

$$\sqrt{D(1)} = \sqrt{1^2 - 2(1) + 9/4}$$

$$= \sqrt{1 - 2 + 9/4}$$

$$= \sqrt{-1 + 9/4} = \sqrt{5/4} = \frac{\sqrt{5}}{2}$$

47.



$$\sin\theta = \frac{x}{1}$$

$$1\sin\theta = x$$

$$\cos\theta = \frac{h}{1}$$

$$1\cos\theta = h \qquad h = \cos\theta$$

height of trapezoid = $\cos\theta$

base of trapezoid = $1 + 2\sin\theta$

area of trap = $\frac{1}{2}h(b_1 + b_2) = \frac{1}{2}\cos\theta(1 + 1 + 2\sin\theta)$

Volume of trough = Area of base X length

$$V = \frac{1}{2}\cos\theta(1 + 1 + 2\sin\theta)(20)$$

$$V = \frac{1}{2}\cos\theta(2 + 2\sin\theta)(20)$$

$$= 20\cos\theta(1 + 1\sin\theta)$$

$$V' = 20\frac{d}{d\theta}\underbrace{\cos\theta}_{u}\underbrace{(1 + \sin\theta)}_{v} = 20\Big[\underset{u}{\cos\theta}\,\underset{v'}{(\cos\theta)} + \underset{v}{(1 + \sin\theta)}\underset{u'}{(-\sin\theta)}\Big]$$

$$V' = 20(\underbrace{\cos^2\theta}_{} - \sin\theta - \sin^2\theta) = 20(1 - \sin^2\theta - \sin\theta - \sin^2\theta)$$

$*\ \cos^2\theta = 1 - \sin^2\theta$ hint

$$= 20(-2\sin^2\theta - \sin\theta + 1)$$

$$0 = -20(2\sin^2\theta + \sin\theta - 1)$$

$$0 = -20(2\sin\theta - 1)(\sin\theta + 1)$$

$2\sin\theta - 1 = 0$ $\quad\quad$ $\sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ $\quad\quad$ $\sin\theta = -1$

$\theta = \frac{\pi}{6}$ $\quad\quad$ $\cancel{\theta = \frac{\pi}{2}} = 90^\circ$ since look @ diagram of Δ

$\theta = \pi/6$

V' $\quad$ $--+$ $\quad$ $-++$

P $\quad$ $\pi/6$ $\quad$ N

When $\theta = \pi/6$ the trough will have a maximum volume since V' goes from $+$ to $-$ @ $\theta = \pi/6$.