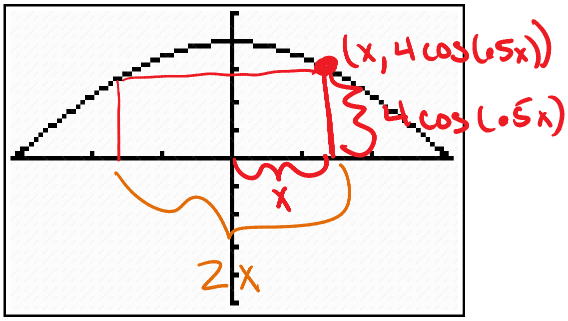
4.4 Day 4 p. 228: 21, 30, 31, 35

Friday, November 23, 2012
10:03 AM

21.



$A = 2x(4\cos(.5x))$

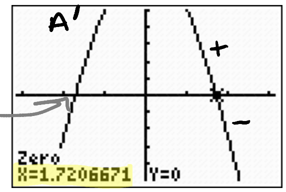
$= 8x\cos(.5x)$

$A' = 8x(-\sin(.5x)(.5)) + 8\cos(.5x))$

$0 = -4x\sin(.5x) + 8\cos(.5x)$

find zero using graphing calc.

we don't use the negative zero!



$x \approx 1.72$

A' + 1.721 −

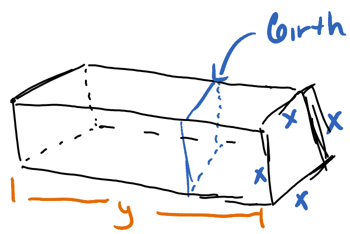
from the graph of A'

Store your zero!!!

max. area is 8.978

☆ The max area is 8.978 when $x \approx 1.721$ b/c A' goes from + to − @ $x \approx 1.721$.

30.



x = the length in inches of each edge of the square end

y = the length of the box

$4x + y \leq 108$ inches since we want to maximize the volume $4x + y = 108$ or $\boxed{y = 108 - 4x}$

$\boxed{\text{Domain } (0, 27)}$

$V = x^2 \cdot y$ or $V = x^2 \cdot (108 - 4x)$

$V = 108x^2 - 4x^3$

$V' = 216x - 12x^2$

$0 = 12x(18 - x)$

C.V are $\cancel{x = 0}$ $x = 18$

out of domain

V' 0 P 18 N 27

max

$y = 108 - 4(18)$
$= 36$

max volume = 11,664 in^3 when x = 18 inches b/c V' goes from + to − @ x = 18 inches

☆ Box dimensions
18in x 18in x 36in

31a. $2x + 2y = 36\text{cm}$ circumference $= x$ $2\pi r = x$

$r = \frac{x}{2\pi}$

$2y = 36 - 2x$

$y = 18 - x$ (height of cylinder)

Domain $(0, 18)$

$V = \pi r^2 h$ $V = \pi \left(\frac{x}{2\pi}\right)^2 (18 - x)$

$V = \pi \left(\frac{x^2}{4\pi^2}\right)(18 - x) = \frac{x^2(18-x)}{4\pi} = \frac{1}{4\pi}(18x^2 - x^3)$

$V' = \frac{1}{4\pi} \cdot \frac{d}{dx}(18x^2 - x^3) = \frac{1}{4\pi}(36x - 3x^2)$

$0 = 36x - 3x^2$

$0 = 3x(12 - x)$

$= \cancel{x=0}$ $x = 12$

V': 0 P 12 N 18

max volume occurs when

$y = 18 - 12$
$= 6\text{cm}$

$x = 12\text{ cm}$ and $y = 6\text{cm}$

b. radius $= x$ height y and $y = 18 - x$

$V = \pi x^2(18 - x) = \pi(18x^2 - x^3)$

$V' = \pi(36x - 3x^2)$

$0 = 36x - 3x^2$ $3x(12 - x^2)$

V': 0 P 12 N 18

$x = 12\text{cm}$

$y = 6\text{cm}$

This is the same as part a.

35. $f(x) = x^3 + ax^2 + bx$

a. $f'(x) = 3x^2 + 2ax + b$ we need $f'(-1) = 0$ & $f'(3) = 0$

$f'(3) = 0$ $0 = 3(3)^2 + 2a(3) + b$

$0 = 27 + 6a + b$

$f'(-1) = 0$ $0 = 3(-1)^2 + 2a(-1) + b$

$0 = 3 - 2a + b$

$0 = 3 - 2a + b$

$-(0 = 27 + 6a + b)$

* subtract equations it's a system

$- (0 = 27 + 6a + b)$ ... it's a system

$0 = -24 - 8a$

$24 = -8a$

$a = -3$

$3 - 2(-3) + b = 0$

$3 + 6 + b = 0$

$9 + b = 0$

$b = -9$

$f(x) = x^3 + ax^2 + bx$ $\quad$ $f(x) = x^3 - 3x^2 - 9x$

we need to verify the local max and local min

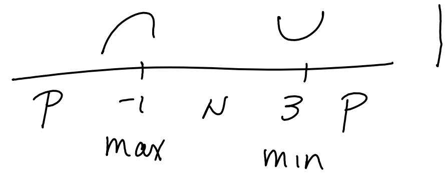
$f'(x) = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$= 3(x-3)(x+1)$

$x = 3 \quad x = -1$

→ f'



$\boxed{☆\ f(x) = x^3 - 3x^2 - 9x}$

There is a local max on $f(x)$ when $x=-1$ b/c $f'(x)$ goes from + to − @ $x=-1$.

There is a local min on $f(x)$ when $x=3$ b/c $f'(x)$ goes from − to + @ $x=3$

b. local min at $x=4$ $\quad$ point of inflection at $x=1$

$f'(x) = 3x^2 + 2ax + b$

$f'(4) = 0$

$3(4)^2 + 2a(4) + b = 0$

$48 + 8a + b = 0$

$f''(x) = 6x + 2a$

$f''(1) = 0$

$0 = 6(1) + 2a$

$0 = 6 + 2a$

→ $-6 = 2a$

$a = -3$

$48 + 8(-3) + b = 0$

$48 - 24 + b = 0$

$b = -24$

$\boxed{☆\ f(x) = x^3 - 3x^2 - 24x}$

we need to verify!!

$f'(x) = 3x^2 - 6x - 24$

$f''(x) = 6x - 6$

Second Derivative Test for local

check $x=4$

$f''(4) = 6(4) - 6$

check p.o.i @ $x=1$

$0 = 6x - 6$

Second Derivative Test for local extrema

check $x=4$

$f''(4)=6(4)-6$

positive

$\therefore$ local min @ $x=4$

check p.o.i @ $x=1$

$0=6x-6$

$6=6x$

$1=x$

f'': N | 1 | P

$\therefore$ there is a p.o.i @ $x=1$