4.4 Day 3 p. 227: 14, 17, 20, 22

Friday, November 23, 2012
8:33 AM

14. $S = -16t^2 + 96t + 112$

a. $v = -32t + 96$ $t = 0$ velocity 96 ft/sec ☆

b. $0 = -32t + 96$ $t = 3$ c.v.

v: P | 3 | N

$S = -16(3)^2 + 96(3) + 112$

$S(3) = 256$ feet

☆ The max. height is 256 ft and occurs @ t=3 because s′ or velocity goes from + to − @ t=3.

c. $0 = -16t^2 + 96t + 112$ $t = 7$

$= -16(t+1)(t-7)$

$v(7) = -32(7) + 96 = -128$ ft/sec ☆

17. $V = 1000\,cm^3$

$\pi r^2 h = 1000$

$h = \frac{1000}{\pi r^2}$

$A = 8r^2 + 2\pi r h$

$= 8r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$= 8r^2 + \frac{2000}{r}$

$\frac{dA}{dr} = 16r - \frac{2000}{r^2}$

$0 = 16r - \frac{2000}{r^2}$

$\frac{2000}{r^2} = 16r$ $r^3 = \frac{2000}{16}$

$r^3 = 125$ $r = 5$ cm

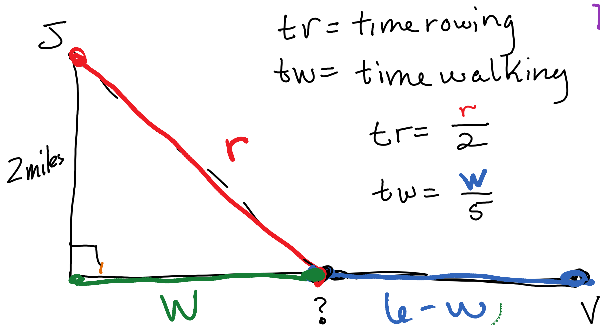
$\frac{dA}{dr}$: 0 | N | 5 | P (∪ min)

$r = 5$ $h = \frac{1000}{\pi(5)^2}$

$h = \frac{40}{\pi}$

☆ Ratio is 5 to $\frac{40}{\pi}$ or 1 to $\frac{8}{\pi}$

20.



t_r = time rowing

t_w = time walking

$t_r = \frac{r}{2}$

$t_w = \frac{w}{5}$

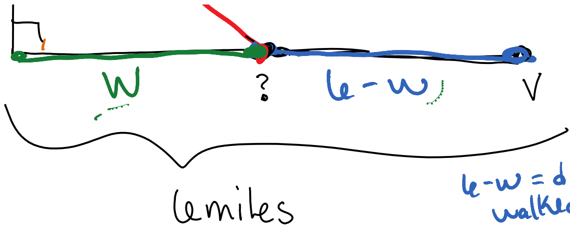
$D = rt$ $t = \frac{D}{r}$

Total Time = time rowing + Time walking

$T = t_r + t_w$

$T = \frac{r}{2} + \frac{6-w}{5}$

$T = \frac{\sqrt{4+w^2}}{2} + \frac{6-w}{5}$



$6 - w$ = distance walked

use the right triangle:

$4 + w^2 = r^2$

$r = \sqrt{4 + w^2}$

$T = \frac{\sqrt{4+w^2}}{2} + \frac{6-w}{5}$

$T' = \frac{1}{4}(4+w^2)^{-1/2} \cdot 2w + -\frac{1}{5}$

$= \frac{w}{2\sqrt{4+w^2}} + {}^{-1}/5$

$+\frac{1}{5} = \frac{w}{2\sqrt{4+w^2}}$ $\quad (+2\sqrt{4+w^2})^2 = (5w)^2$

$4(4+w^2) = 25w^2$

$16 + 4w^2 = 25w^2$

$16 = 21w^2$

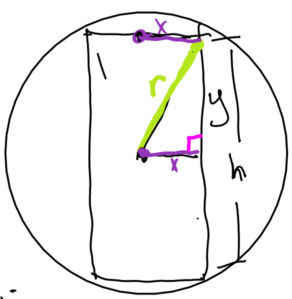
$w^2 = \frac{16}{21}$

$w = \sqrt{16/21} = \frac{4}{\sqrt{21}} \approx .873$

T': N | .873 | P

Jane should land her boat .873 miles down the shoreline to reach the village in the least amount of time.

22.



The cylinder & sphere
* share the same center

$\{$ x = radius of the cylinder
$\{$ r = radius of the sphere

Volume of the cylinder = $\pi r^2 h$

$r = 10$ (given)

$y = \frac{1}{2}$ of the height of the cylinder

$\therefore y = \frac{1}{2}h$

$x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

$y^2 = 10^2 - x^2$

$y = \sqrt{100 - x^2}$

$\frac{1}{2}h = \sqrt{100 - x^2}$

$h = 2\sqrt{100 - x^2}$

$V = \pi x^2 h$

$V = \pi x^2 (2\sqrt{100 - x^2})$

$= 2\pi x^2 \sqrt{100 - x^2}$

$V' = 2\pi\ x^2 \left(\frac{1}{2}(100 - x^2)^{-1/2}(-2x)\right) + \sqrt{100 - x^2}\,(4\pi r)$

$V' = \frac{-2x\,\pi x^2}{\sqrt{100 - x^2}} + 4\pi x(\sqrt{100 - x^2})$

$0 = \frac{-2x\pi x^2 + 4\pi x(100 - x^2)}{\sqrt{100 - x^2}}$

use calc. to find zero $\quad x \approx 8.165$

V': 0 | P | 8.165 max | N | 10

max volume $\approx$ 2418.399 cm^3

The maximum volume of the cylinder is 2418.399 cm^3 when x (radius) is 8.165 cm because 'v' goes from + to − at x = 8.165 cm.