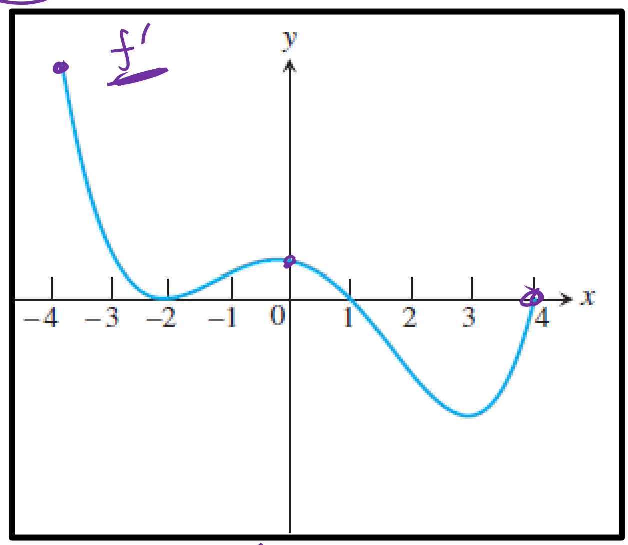
AP Calc AB
Notes 5.3 Day 2

Name Key

The graph, to the right, is the derivative of a function f on the interval $[-4,4]$.



a. On what intervals is f increasing?

f is increasing $(-4,1)$ b/c $f' > 0$ on the interval $(-4,-2) \cup (-2,1)$.

b. On what intervals is the graph of f concave up?

when f' is increasing

f is ccu $(-2,0) \cup (3,4)$ b/c f' is increasing on the interval.

f': P P N

-4 -2 1 4

c. At which x-coordinates does f have local extrema?

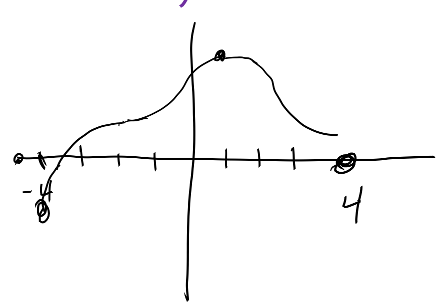
- $f(x)$ has a local max @ $x=1$ b/c f' goes from + to − @ $x=1$.
- $f(x)$ has a local min @ $x=-4$ b/c @ $x=-4$ there is an endpt and $f'>0$ to the right of $x=-4$,
- $f(x)$ has a local min @ $x=4$, b/c @ $x=4$ there is an endpt and $f'<0$ to the left of $x=4$.

d. What are the x-coordinates of all inflection points of the graph f?

f has a p.o.i @ $x=-2$, $x=0$, $x=3$

b/c f' goes from either increasing to decreasing or decreasing to increasing @ $x=-2$, $x=0$, $x=3$

e. Sketch a possible graph of f on the interval $[-4,4]$.



THEOREM 5 Second Derivative Test for Local Extrema

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
2. If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

Using the second derivative test, find all local extreme values:

a. $f(x) = x^3 - 12x - 5$

$f'(x) = 3x^2 - 12$

$3x^2 - 12 = 0$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

$f''(x) = 6x$

$f''(2) = 12$

$f''(-2) = -12$

$x = 2$ local min

$x = -2$ local max

Your turn.....

Given $f(x) = x^3 + \frac{3}{2}x^2 - 6x + 1$

a. Find all extrema on $f(x)$ using the SECOND Derivative test. Justify your answer.

$f'(x) = 3x^2 + 3x - 6$

$= 3(x^2 + x - 2)$

$\rightarrow 0 = 3(x+2)(x-1)$

$x = -2 \quad x = 1$

$f'' = 6x + 3$

$f''(-2) < 0$

$f(1) > 0$

$x = -2$ local max

$x = 1$ local min

b. Determine the concavity of the graph of $f(x)$. Justify your answer.

$f''(x) = 6x + 3 \qquad 6x + 3 = 0$

f'' N $-\frac{1}{2}$ P

f(x) is ccu $(-\frac{1}{2}, \infty)$ b/c $f'' > 0$ on the interval.

f(x) is ccd $(-\infty, -\frac{1}{2})$ b/c $f'' < 0$ on the interval

c. Determine all points of inflection on the graph of $f(x)$. Justify your answer.

$(-.5, 4.25)$

f(x) has a p.o.i $(-.5, 4.25)$ b/c f'' changes signs @ $x = -.5$.

d. Sketch a possible graph of $f(x)$.