5.3 Day 2 (Monday 10/7)

Friday, October 4, 2019 10:25 AM

AP Calc BC
Notes 5.3 Day 2

Name______________________________

RECALL:

THEOREM 5 Second Derivative Test for Local Extrema

① If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.

② If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

Using the second derivative test, find all local extreme values. Justify your answer.

$f(x) = x^3 - 12x - 5$

$f' = 3x^2 - 12$

$0 = 3x^2 - 12$

① $x = \pm 2$

$f'' = 6x$

② $f''(2) = 6(2) > 0$ local min

$f''(-2) = 6(-2) < 0$ local max

f(x) has a local min @ $x = 2$, b/c $f'(2) = 0$ & $f''(2) > 0$.

f(x) has a local max @ $x = -2$, b/c $f'(-2) = 0$ & $f''(-2) < 0$.

Find all the **points** of inflection on the graph $f(x)$ and the intervals of concavity on the graph of $f(x)$. Justify.

$f(x) = e^{-x^2}$

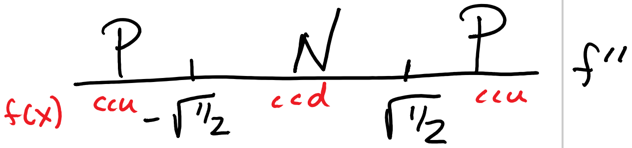
$f'(x) = e^{-x^2} \cdot (-2x) = -2x e^{-x^2}$

$f''(x) = -2x\left(e^{-x^2} \cdot (-2x)\right) + e^{-x^2}(-2)$

$= 4x^2 e^{-x^2} - 2e^{-x^2}$

$0 = 2e^{-x^2}(2x^2 - 1)$

$x = \pm\sqrt{1/2}$



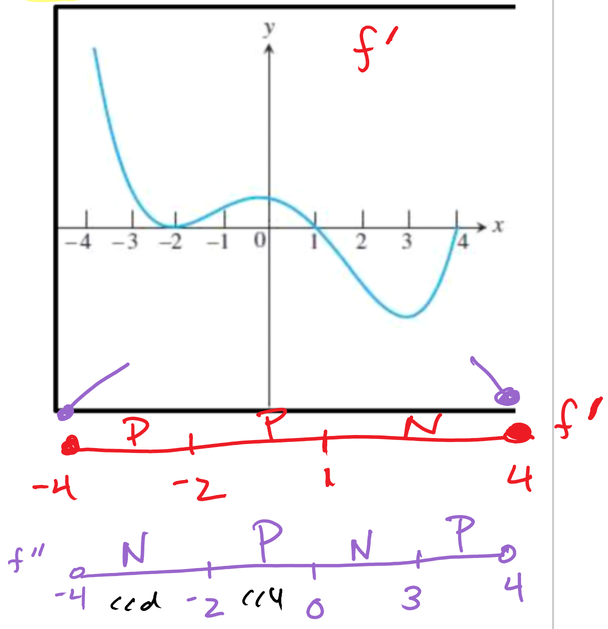
★ f(x) has p.o.i.s @ $x = \pm\sqrt{1/2}$
b/c f'' changes signs @ $x = \pm\sqrt{1/2}$.

★ f(x) is ccu $(-\infty, -\sqrt{1/2}) \cup (\sqrt{1/2}, \infty)$
b/c $f'' > 0$ over the intervals.

★ f(x) is ccd $(-\sqrt{1/2}, \sqrt{1/2})$, b/c $f'' < 0$ over the interval.

The graph, to the right, is the derivative of a function f on the interval $[-4,4]$.

closed interval



a. On what intervals is f increasing?

$(-4, 1)$

$(-4,-2)\cup(-2, 1)$

b. On what intervals is the graph of f concave up?

f is ccu $(-2,0)\cup(3,4)$
b/c f' is increasing over
the interval.

c. At which x-coordinates does f have local extrema?

$x=-4$ local min

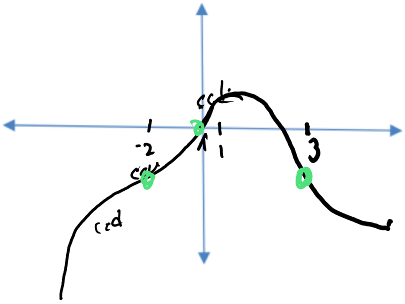
$x=1$ local max

$x=4$ local min

d. What are the x-coordinates of all inflection points of the graph f?

$x=-2, 0, 3$

e. Sketch a possible graph of f on the interval $[-4,4]$.



Your turn.....

Given $f(x) = x^3 + \frac{3}{2}x^2 - 6x + 1$

a. Find all extrema on $f(x)$ using the SECOND Derivative test. Justify your answer.

$f'(x) = 3x^2 + 3x - 6$

$= 3(x^2 + x - 2)$

$= 3(x+2)(x-1)$

$f''(x) = 6x+3$

$f'(1) = 0$

$f''(1) > 0$

$f'(-2) = 0$

$f''(-2) < 0$

- f(x) has a local max @ $x = -2$ b/c $f'(-2) = 0$ & $f''(-2) < 0$.
- f(x) has a local min @ $x = 1$ b/c $f'(1) = 0$ & $f''(1) > 0$.

b. Determine the concavity of the graph of $f(x)$. Justify your answer.

$f'' = 6x+3$

$0 = 6x+3$

f'' N P

$-\frac{1}{2}$

- f(x) ccd from $(-\infty, -\frac{1}{2})$ b/c $f'' < 0$ over the interval
- f(x) ccu $(-\frac{1}{2}, \infty)$ b/c $f'' > 0$ over the interval.

c. Determine all points of inflection on the graph of $f(x)$. Justify your answer.

f(x) has a p.o.i of $(-0.5, 4.25)$ b/c $f''(-\frac{1}{2}) = 0$ & f'' changes signs @ $x = -\frac{1}{2}$.

d. Sketch a possible graph of $f(x)$.