5.3 Day 1

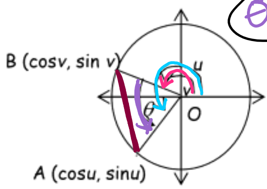
Tuesday, April 2, 2019 3:05 PM

Pre Calc Honors

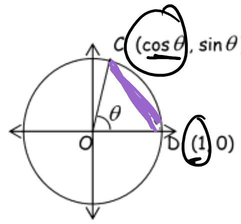
Sec 5.3 Sum and Difference Identities

Name______________________________

$\theta = u - v$



Angles u and v are drawn in standard position in the unit circle on the left. They determine two points on the circle, A and B, with coordinates as indicated. In the unit circle on the right, ΔAOB has been rotated so that the angle $\theta = u - v$ is in standard position. The angle θ determines point C with coordinates as indicated. Since ΔAOB and ΔCOD are congruent, AB = CD.



If we use the distance formula to find AB and CD, we have:

$$\sqrt{(\cos v - \cos u)^2 + (\sin v - \sin u)^2} = \sqrt{(\cos\theta - 1)^2 + (\sin\theta - 0)^2}$$

Solve this equation for $\cos\theta$.

$$(\cos v - \cos u)^2 + (\sin v - \sin u)^2 = (\cos\theta - 1)^2 + \sin^2\theta$$

$$\cos^2 v - 2\cos v\cos u + \cos^2 u + \sin^2 v - 2\sin v\sin u + \sin^2 u = \cos^2\theta - 2\cos\theta + 1 + \sin^2\theta$$

$$\cancel{2} - 2\cos v\cos u - 2\sin v\sin u = \cancel{2} - 2\cos\theta \Rightarrow \cos v\cos u + \sin v\sin u = \cos\theta$$

Since $\theta = u - v$, $\cos(u - v) = \boxed{\cos v\cos u + \sin v\sin u}$

Use the results above to find $\cos(u + v)$. (Hint: $\cos(u + v) = \cos(u - (-v))$ so use the Odd-Even Identities!)

$$\cos(u+v) = \cos(u-(-v)) = \cos u \cdot \cos(-v) + \sin(u)(\sin(-v))$$

$$\boxed{\cos(u+v) = \cos u\cos v - \sin(u)\sin(v)}$$

Summarize the two identities you just discovered.

<u>Cosine of a Sum or Difference</u>

$$\cos(u \pm v) = \cos u\cos v \mp \sin u\sin v$$

$$\cos(u+v) = \cos u\cos v - \sin u\sin v \qquad \cos(u-v) = \cos u\cos v + \sin u\sin v$$

$$\theta = u+v$$

Ok, you're doing pretty well here. Keep going! Use the Cofunction and Odd-Even Identities to find the identities for sin(*u* - *v*) and sin(*u* + *v*).

$$\sin\theta = \cos\left(\tfrac{\pi}{2}-\theta\right) = \cos\left(\tfrac{\pi}{2}-(u+v)\right) = \cos\left(\left(\tfrac{\pi}{2}-u\right)+(-v)\right)$$

$$\cos\left(\tfrac{\pi}{2}-u\right)\cos(-v) - \sin\left(\tfrac{\pi}{2}-u\right)\sin(-v)$$

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

Now summarize the <u>Sine of a Sum or Difference</u>.

$$\sin(u \pm v) = \sin u \cos v \pm \sin v \cos u$$

All that's left now is to determine the identities for the <u>Tangent of a Sum or Difference.</u> Derive those below, writing your final answer only in terms of the tangent function.

$$\tan(u \pm v) = \frac{\sin(u \pm v)}{\cos(u \pm v)} = \frac{\sin u \cos v \pm \sin v \cos u}{\cos u \cos v \mp \sin u \sin v}$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

example: Evaluate a. $\cos(15^\circ)$ Ref 30° 45° 60° 90°

$$\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$\frac{\sqrt2}{2}\cdot\frac{\sqrt3}{2} + \frac{\sqrt2}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt6}{4} + \frac{\sqrt2}{4} = \boxed{\frac{\sqrt6+\sqrt2}{4}}$$

b. $\sin(165^\circ) = \sin(210 - 45) = \sin 210 \cos 45 - \sin 45 \cos 210$

$$= -\frac{1}{2}\cdot\frac{\sqrt2}{2} - \frac{\sqrt2}{2}\cdot -\frac{\sqrt3}{2}$$

$$= -\frac{\sqrt2}{4} + \frac{\sqrt6}{4} = \frac{-\sqrt2+\sqrt6}{4}$$

c. $\tan\left(\frac{7\pi}{12}\right)$

$\frac{\pi}{2}$ $\frac{\pi}{3}$ $\frac{\pi}{4}$ $\frac{\pi}{6}$

$\frac{6\pi}{12}$ $\frac{4\pi}{12}$ $\frac{3\pi}{12}$ $\frac{2\pi}{12}$

$$\tan\left(\frac{4\pi}{12}+\frac{3\pi}{12}\right) = \tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

$\tan\frac{\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$$\frac{\sqrt{3}+1}{1-\sqrt{3}}$$

$\sin(u+v) = \sin u\cos v + \cos u \sin v$

$y = \sin x\cos 1 + \cos x \sin 1$

$y = \sin(x+1)$