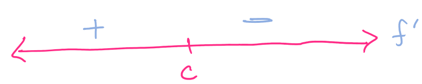
5.3 Day 1

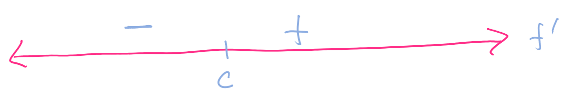
Friday, October 4, 2019 8:11 AM

1st Derivative Test → used to test & justify a point is a local/relative max or min.

☆ $x=c$ is a critical point of $f(x)$, then:

1) 

f has a local max @ $x=c$ since f' changed from + to − @ $x=c$.

2) 

f has a local min @ $x=c$ since f' changed from − to + @ $x=c$

At left endpoint $x=a$:

1) 

f has a local min @ $x=a$ since $f'>0$ to the right of $x=a$.

2) 

f has a local max @ $x=a$ since $f'<0$ to the right of $x=a$.

At right endpoint, $x=b$:

1) 

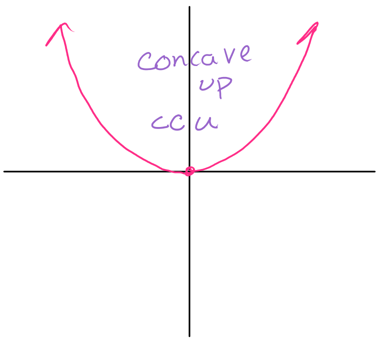
f has a local max @ $x=b$ since $f'>0$ to the left of $x=b$.

2) 

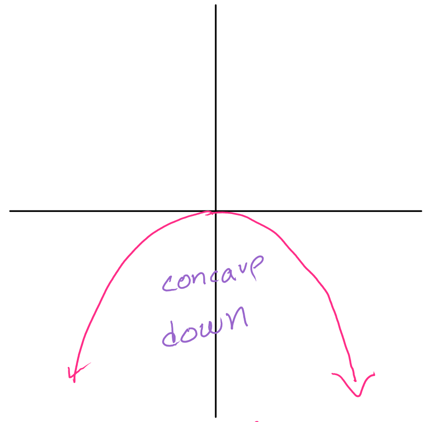
f has a local min @ $x=b$ since $f'<0$ to the left of $x=b$.

graph:

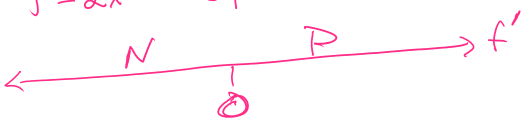
$f(x) = x^2$



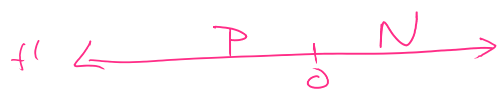
$f(x) = -x^2$



$f' = 2x$ c.p. $x = 0$



$f' = -2x$ c.p $x = 0$



$f'' = 2$

$f'' = -2$

The 2^{nd} derivative tells us the concavity of the graph of f.

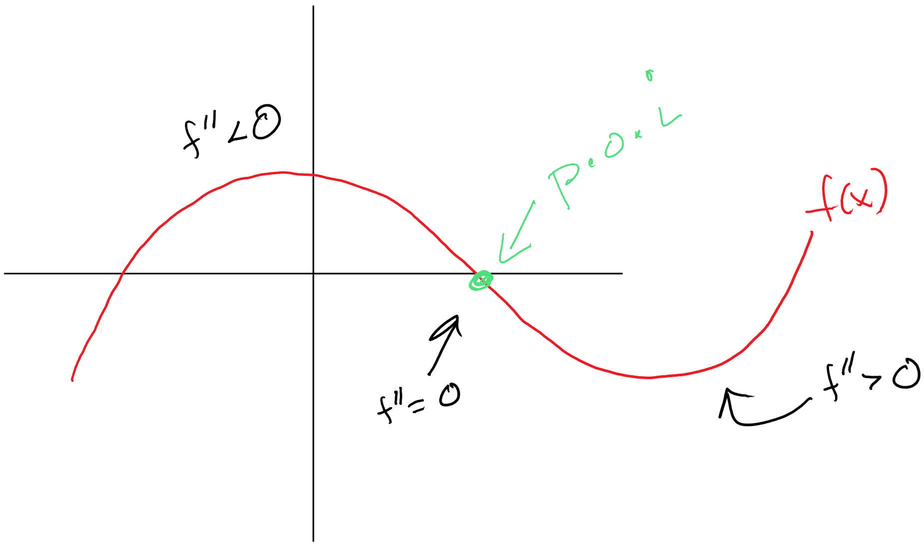
$f'' > 0$ then f is ccu "like a cup"

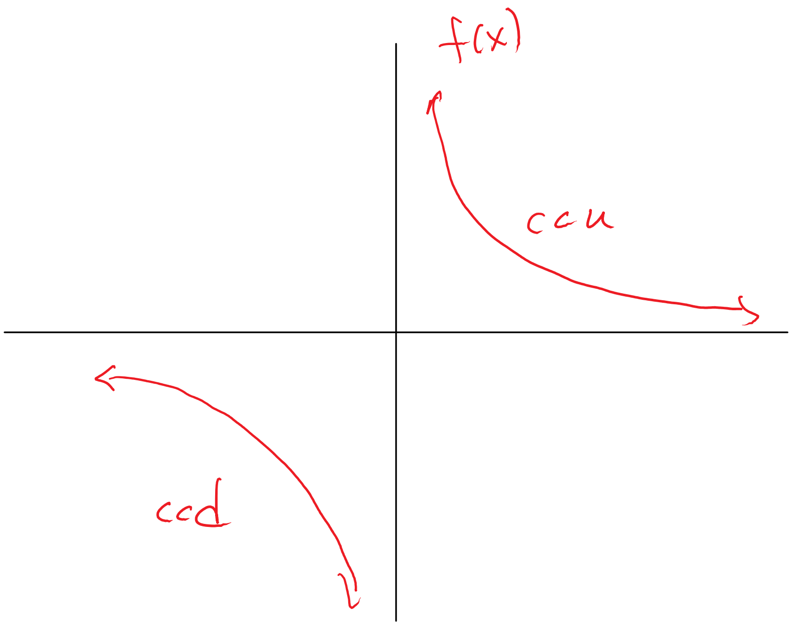
$f'' < 0$ then f is ccd "like a frown"

☆ f'' changes signs @ $x = a$

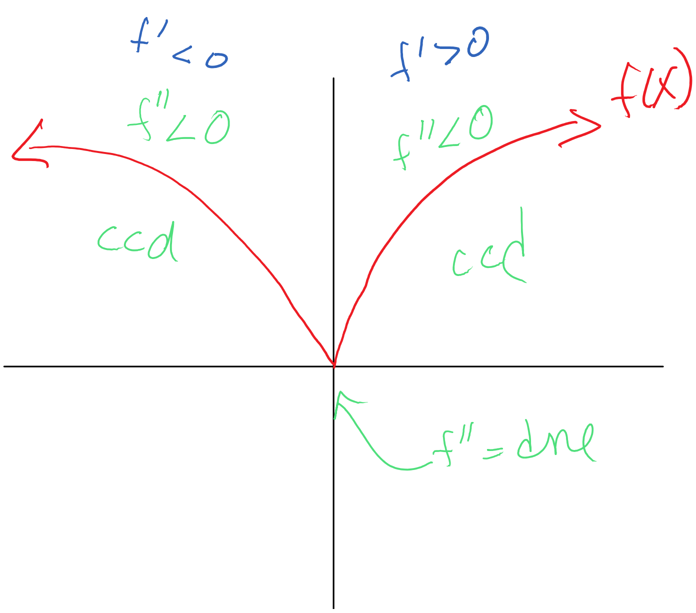
$f''(a) = 0$ or $f''(a) = $ dne & $f(a)$ exists on $f(x)$

then $(a, f(a))$ is the Point of Inflection (p.o.i)



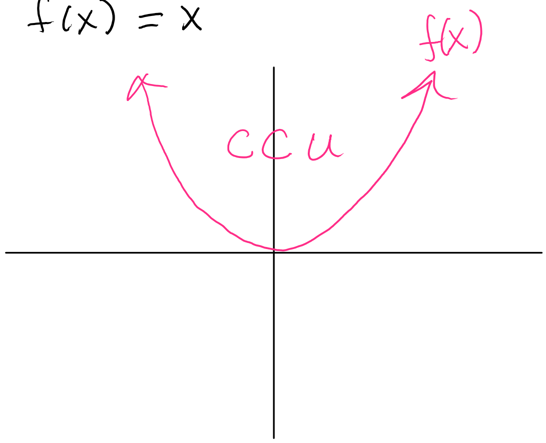


NO POI



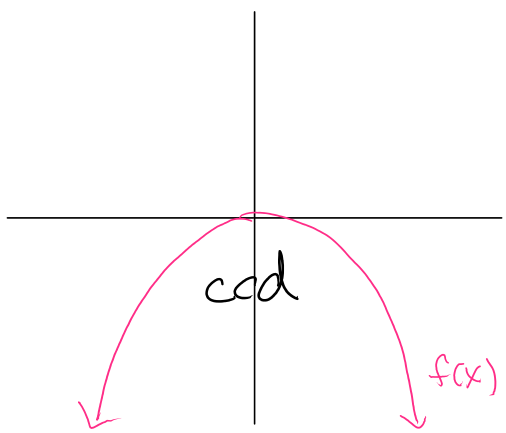
2nd Derivative Test

$f(x) = x^2$



$f'(0) = 0$ $\quad x=0$ c.p.

$f''(0) > 0$ $\quad f(x)$ ccu



$f'(0) = 0$ $\quad x=0$ c.p.

$f''(0) < 0$ $\quad f(x)$ ccd

1. If $f'(c)=0$ and $f''(c)<0$, then f has a local max @ $x=c$.

2. If $f'(c)=0$ and $f''(c)>0$, then f has

a local min.

Ex: Use 2^{nd} Derivative test to find all local extrema. Justify.

$$f(x) = 2x^4 - 4x^2 + 3$$

$f'(x) = 8x^3 - 8x$ $\quad 8x(x^2-1) = 0$

$x = 0 \quad x = \pm 1$

$f''(x) = \underline{24x^2} - 8$

Test

$x=0 \quad f''(0) < 0$

$f(x)$ has a local max, b/c $f'(0)=0$ & $f''(0)<0$,

$x=1 \quad f''(1) > 0$

$f(x)$ has a local min, b/c $f'(1)=0$ & $f''(1)>0$

$x=-1 \quad f''(-1) > 0$

$f(x)$ has a local min, b/c $f'(1)=0$ & $f''(-1)>0$.