AP Calculus AB Name____________________

5.3-5.4 Review

Things to think about:

- The first derivative test (sign chart with written justification) is used to find where local max and local mins occur on f(x).
- When finding absolute extrema you MUST make a candidate chart (this includes any critical points and endpoints). It is essentially a table of values for f(x) that gives the output on f(x) for all endpoints and critical points.
- ALWAYS check endpoints!!!! They are most likely either a local/absolute max or min.
- The first derivative of a function ($f'(x)$) tells us where the function ($f(x)$) is increasing or decreasing.
- The second derivative test can be used to determine local max/mins.
- The second derivative of a function ($f''(x)$) tells us where a point of inflection occurs on $f(x)$ and the concavity of the graph of $f(x)$.

1. Find the absolute minimum of $f(x) = x^3 - 5x^2 - 8x + 2$ over [-2,7]. Justify your answer!

$f'(x) = 3x^2 - 10x - 8$

$0 = (3x+2)(x-4)$

f′: P on (−2, −2/3), N on (−2/3, 4), P on (4, 7)

f(x): increasing, max at $-2/3$, min at 4, increasing

Candidate chart

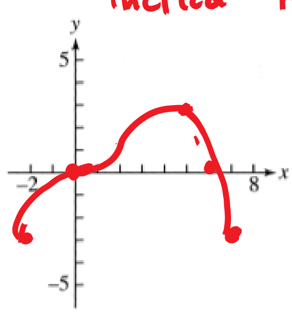
x	f(x)
-2	-10
-2/3	4.8148
4	-46 *
7	44

f(x) has an abs min of -46 @ x=4, b/c f′(x) goes from − to + @ x=4.

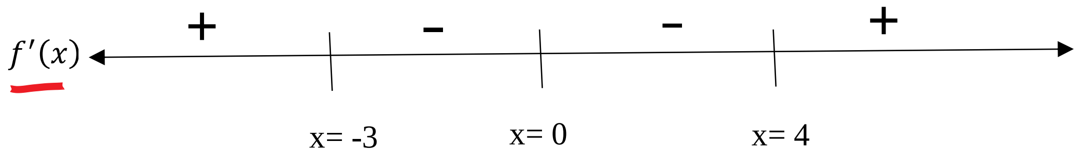
2.

Sketch a graph of a differentiable function $f(x)$ over the closed interval [−2, 7], where $f(-2) = f(7) = -3$ and $f(4) = 3$. The roots of $f(x) = 0$ occur at $x = 0$ and $x = 6$, and $f(x)$ has properties indicated in the table below:

x	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x = 2$	$2 < x < 4$	$x = 4$	$4 < x < 7$
$f'(x)$	positive	0	positive	1	positive	0	negative
$f''(x)$	negative	0	positive	0	negative	0	negative
	inc/ ccd	p.o.i	inc ccu	p o i	inc ccd	↑ local max	dec ccd



3. Given $f(x)$ is a twice differentiable function, use the sign chart of $f'(x)$ below to answer the following questions.



a. State where $f(x)$ is increasing and justify your answer.

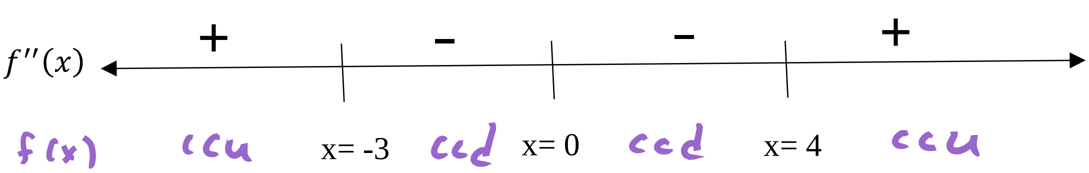
f(x) is increasing $(-\infty,-3)\cup(4,\infty)$ b/c $f'(x)>0$ over the interval.

b. State where all local extrema on $f(x)$ will occur, and justify your answer.

f(x) has a local max @ $x=-3$, b/c $f'(x)$ goes from + to - @ $x=-3$.

f(x) has a local min @ $x=4$, b/c $f'(x)$ goes from - to + @ $x=4$.

4. Given $f(x)$ is a twice differentiable function, use the sign chart below of $f''(x)$ *and the fact that* $f'(-6) = 0$ to answer the following questions.



a. State where all local extrema on $f(x)$ will occur, and justify your answer.

f(x) has a local min @ $x=-6$, b/c $f'(-6)=0$ and $f''(-6)>0$.

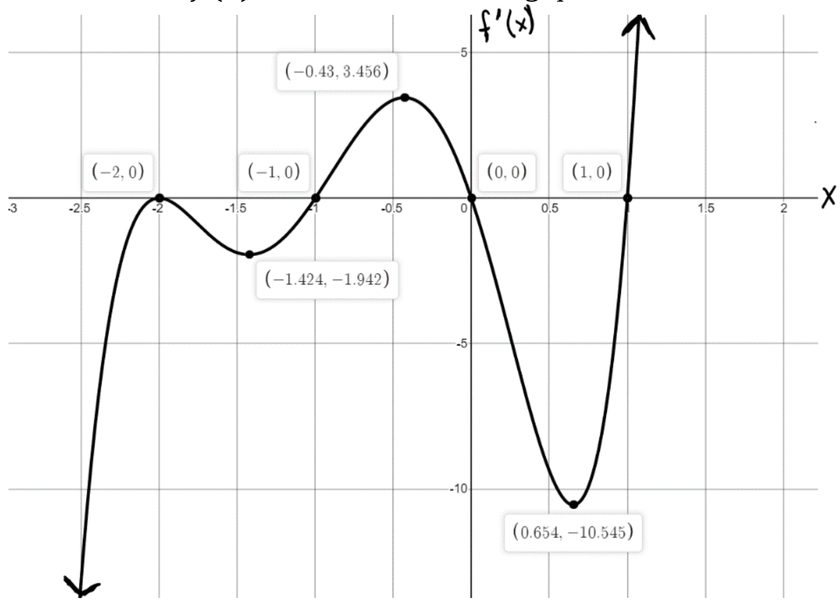
b. State where all points of inflections occur on $f(x)$ and Justify your answer.

f(x) has p.o.i. @ $x=-3$ & $x=4$ b/c $f''(x)$ changes signs @ $x=-3$ and $x=4$.

c. State where the graph of $f(x)$ is concave up and justify your answer.

f(x) is ccu $(-\infty,-3)\cup(4,\infty)$ and $f''(x)>0$ over the interval.

5. Given the graph of the derivative of $f(x)$, answer the following questions.



a. Determine where all the local minimum occurs on $f(x)$. Justify your answer.

f(x) has a local min @ x = −1 & 1 b/c f'(x) goes from − to + @ x = −1 & 1.

b. State where the graph of $f(x)$ is concave down. Justify your answer.

f(x) is ccd (−2, −1.424) ∪ (−0.43, 0.654) b/c f'(x) is decreasing over the interval.

c. How many points of inflection will the graph of $f(x)$ have?

f(x) will have 4 p.o.i.

6. For all the local maximums for $f(x) = e^{x^2-x}$ over the interval [-1,3]. Justify your answer.

$f'(x) = e^{x^2-x}(2x-1)$

f(x) ↘ ∨ ↗

f' N P

−1 $\frac{1}{2}$ 3

$f'(\frac{1}{2}) = 0$

f(x) will have a local max @ x = −1, b/c x = −1 is an endpt and f'(x) < 0 to the right of x = −1.

f(x) has a local max @ x = 3 b/c x = 3 is an endpt and f'(x) > 0 to the left of x = 3.