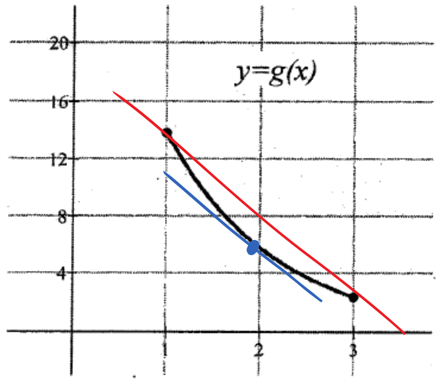
5.2 Mean Value Theorem

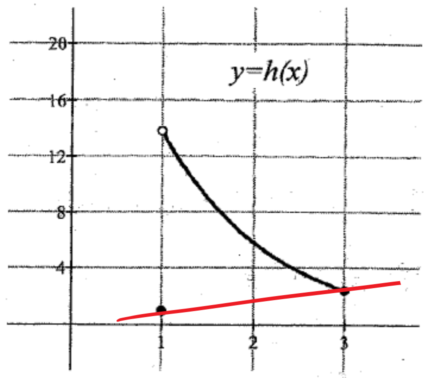
Draw the secant line connecting the endpoints of the given interval for each curve. Decide how many tangent lines can be drawn that are parallel to the secant line.

1. $g(x)$ on the interval $[1, 3]$



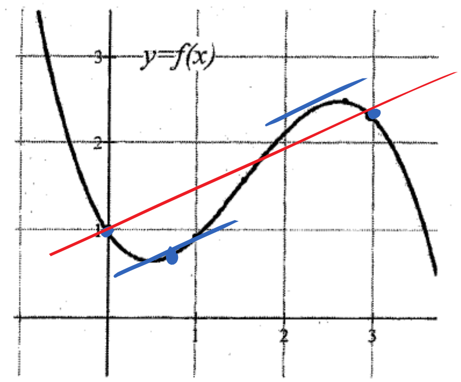
Number of tangents in (1, 3) 1

2. $h(x)$ on the interval $[1, 3]$



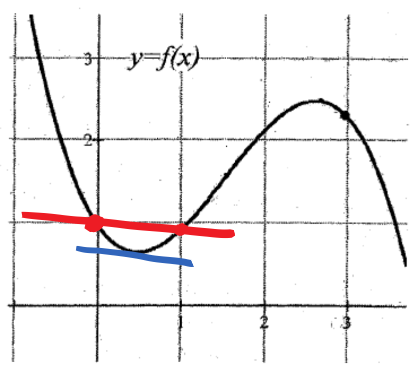
Number of tangents in (1, 3) 0

3. $f(x)$ on the interval $[0, 3]$



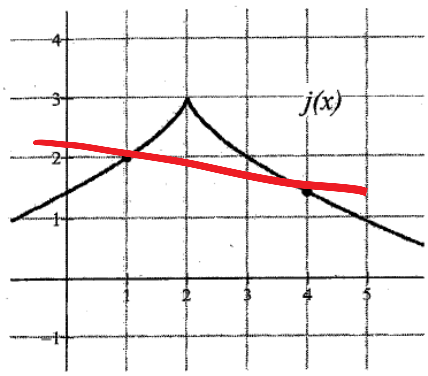
Number of tangents in (0, 3) 2

4. $f(x)$ on the interval $[0, 1]$



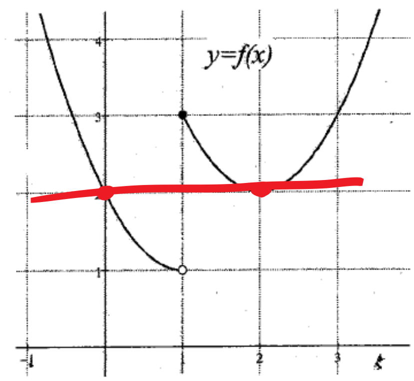
Number of tangents in (0, 1) 1

5. $j(x)$ on the interval $[1, 4]$



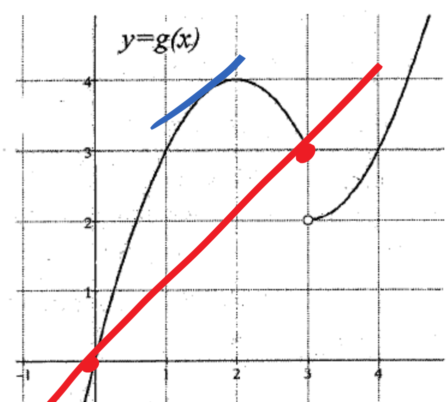
Number of tangents in (1, 4) 0

6. $f(x)$ on the interval $[0, 2]$



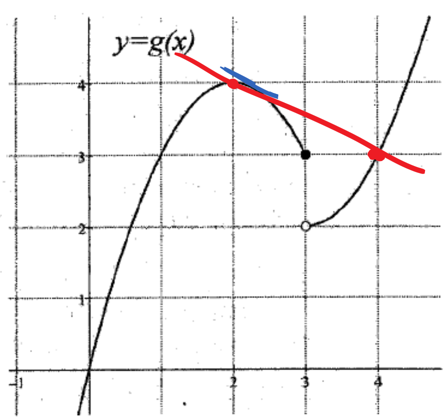
Number of tangents in (0, 2) 0

7. $g(x)$ on the interval $[0, 3]$



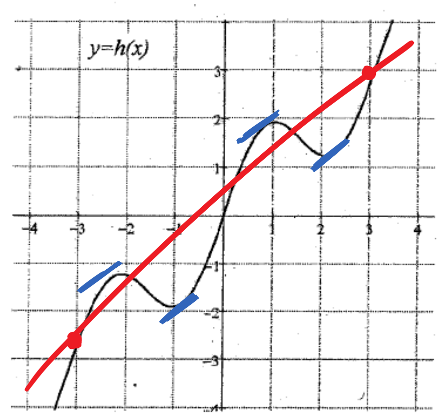
Number of tangents in (0, 3) 1

8. $g(x)$ on the interval $[2, 4]$



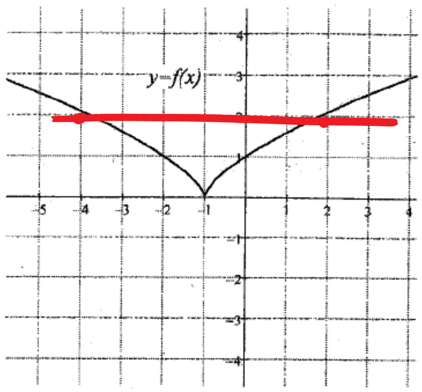
Number of tangents in (2, 4) 1

9. $h(x)$ on the interval $[-3, 3]$



Number of tangents in $(-3, 3)$ 4

10. $f(x)$ on the interval $[-4, 2]$



Number of tangents in $(-4, 2)$ 0

Answer the following questions.

1. Which graphs are continuous on the indicated interval [a, b]?

   1, 3, 4, 5, 7, 9, 10

2. Which graphs are not continuous on the indicated interval [a, b]?

   2, 6, 8

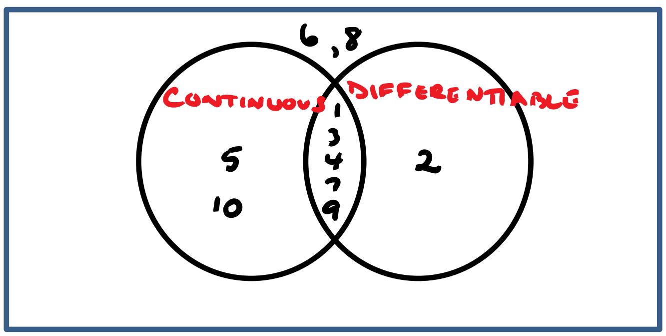
3. Which graphs are differentiable on the open interval (a, b)?

   1, 2, 3, 4, 7, 9

4. Which graphs are not differentiable on the open interval (a, b)?

   5, 6, 8, 10

Venn Diagram



5. If a function is continuous on a closed interval [a, b], is there a tangent line that is parallel to the secant line through the points with x-coordinates x = a and x = b?

   Not necessarily; see Ex 5 and 10

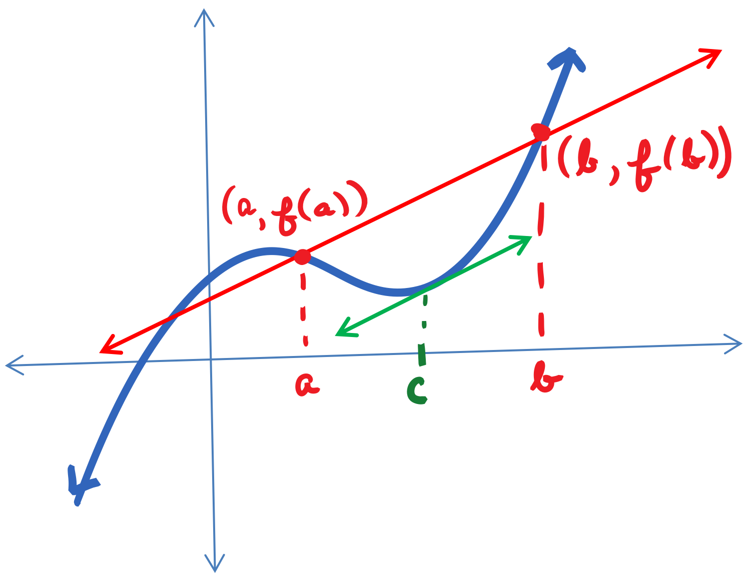
6. If a function is differentiable on an open interval (a, b), is there a tangent line that is parallel to the secant line through the points with x-coordinates x = a and x = b?

   Not necessarily; see Ex 2

7. In the examples above, in order to ensure that the graph of a function has a tangent line that is parallel to the secant line, the function must be continuous on [a, b] and differentiable on (a, b). For which functions do these two conditions hold?

   1, 3, 4, 7, and 9

The Mean Value Theorem geometrically:



Slope of secant line connecting x = a to x = b equals slope of the tangent at c

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

The Mean Value Theorem (MVT) for Derivatives

If $y = f(x)$ is continuous at every point of the closed interval $[a, b]$ and differentiable at every point of the open interval (a, b), then there is at least one point c in (a, b) at which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Note: There may be more than one value for c in the interval.

There may be values of c that fall outside the interval.

For each function and interval, determine if the Mean Value Theorem applies. If it does apply, find all values c where the slope of the tangent lines is equal to the slope of the secant line connecting the endpoints of the given interval.

1. $g(x) = 4x^3 - x^2 + 4$ [-1, 1]

$g'(x) = 12x^2 - 2x$

$$\frac{f(1) - f(-1)}{1 - -1} = f'(c)$$

$$\frac{7 - -1}{2} = 12c^2 - 2c$$

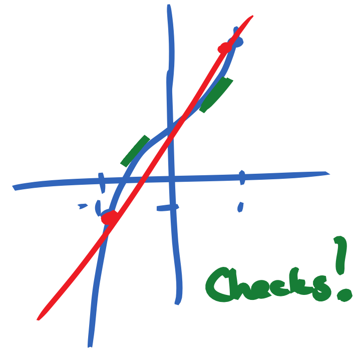
$$4 = 12c^2 - 2c$$

$$0 = 12c^2 - 2c - 4$$

$$0 = 2(6c^2 - c - 2)$$

$$0 = 2(3c - 2)(2c + 1)$$

$\boxed{c = 2/3,\ -1/2}$ (both in interval)



2. $f(x) = \dfrac{1}{x-1}$ [0, 3]

f has an infinite disc. @ x = 1, so f is not diff. (0,3)

M.V.T does not apply

3. $f(x) = \dfrac{1}{x+1}$ [0, 3]

MVT applies!

$f(x) = (x+1)^{-1}$

$$f'(x) = \frac{-1}{(x+1)^2} \cdot 1$$

OR

Quotient Rule

$$f'(x) = \frac{(x+1) \cdot 0 - 1(1)}{(x+1)^2}$$

$$= \frac{-1}{(x+1)^2}$$

$$\frac{f(3) - f(0)}{3 - 0} = f'(c)$$

$$\frac{\frac{1}{4} - 1}{3} = \frac{-1}{(c+1)^2}$$

$$-\frac{1}{4} = \frac{-1}{(c+1)^2}$$

$$-(c+1)^2 = -4$$

$$(c+1)^2 = 4$$

$c + 1 = 2$, $c + 1 = -2$

$\boxed{c = 1}$, $c = -3$

not interval