5.1 Day 1 (Monday 9/30)

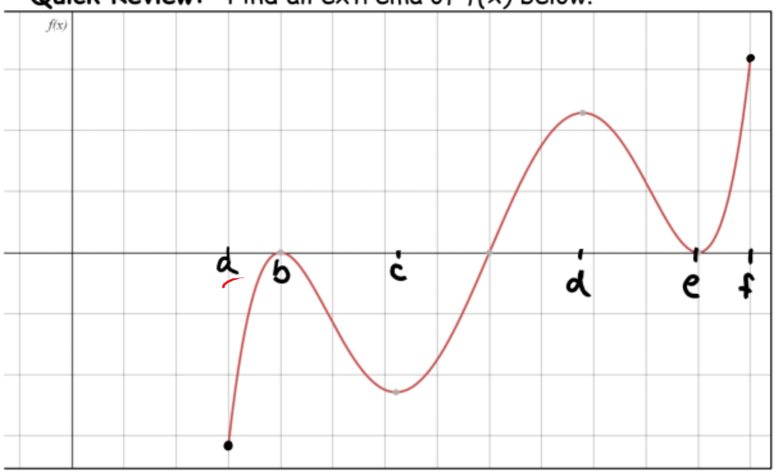
Sunday, September 29, 2019 1:34 PM

Section 5.1 Notes
Extreme Values of a Function

Local | Relative

Absolute | Global

Quick Review: Find all extrema of $f(x)$ below.



Abs min of $f(a)$ @ $x = a$

Local max of $f(b)$ @ $x = b$

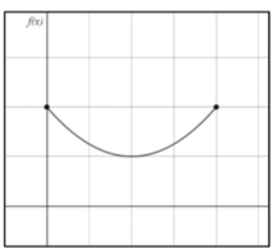
Local min of $f(c)$ @ $x = c$

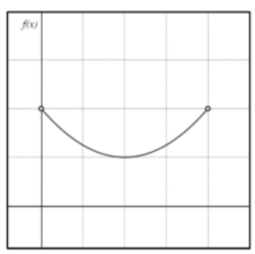
Local max of $f(d)$ @ $x = d$

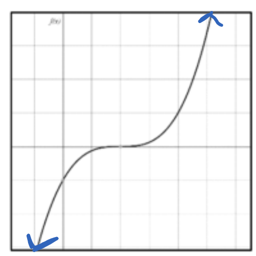
Local min of $f(e)$ @ $x = e$

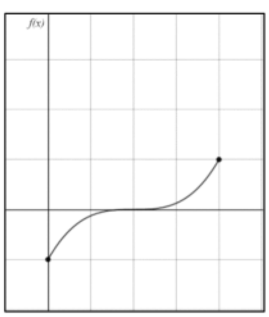
Abs max of $f(f)$ @ $x = f$

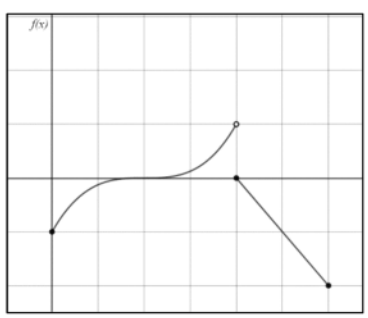
What qualities of a function guarantee that it will have an absolute max AND an absolute min? With your partner, circle which functions below have an abs. max AND an abs. min.

A) 

B) 

C) 

D) 

E) 

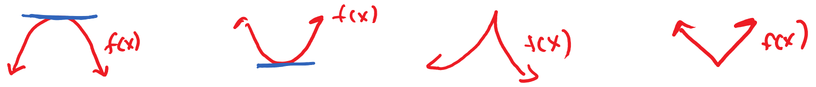
What do the graphs you circled above have in common? Why do the other functions not have an abs. max AND min?

- continuous on a closed interval

Extreme Value Theorem (EVT):

If f is continuous on $[a,b]$, then f has BOTH an abs. max AND an abs. min.

How can we use calculus to find local and absolute extrema?



When $f'(c)=0$ OR $f'(c)=DNE$, then $x=c$ could be a local max or local min.

★ we need to test points around $x=c$ to make sure there is a sign change.

Definition of Critical Point:

A point in the interior of a function with $f'=0$ or $f'=dne$ is called a critical point (c.p.) of f.

Note: You must always check ENDPOINTS when finding extrema! They could be maxs and/or mins too!

Ex: Find the extrema of $f(x) = x^{\frac{2}{3}}$ on [-2, 3] analytically. Justify your answers.

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

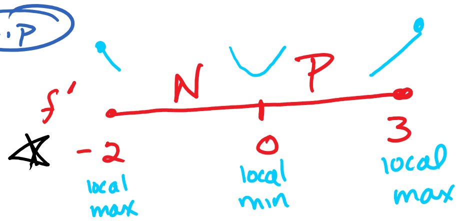
$f'=0$

NOPE

$f'=DNE$

$x=0$ c.p

C.P. $x=0$ Endpts $x=-2, 3$



★

x	f(x)
-2	$(-2)^{2/3}$
0	0
3	$3^{2/3}$ Abs max

f has an Abs min of 0 @ $x=0$, b/c $f'(x)$ changes from − to + @ $x=0$.

f has an Abs max of $3^{2/3}$ @ $x=3$ b/c $x=3$ is an endpt. $f'(x)>0$ to the left of $x=3$.

f has a local max of $(-2)^{2/3}$ @ $x=-2$, b/c $x=-2$ is an endpt & $f'(x)<0$ to the right of $x=-2$.

Ex: Find all extrema of $f(x) = x^3 - 6x^2 + 1$ on [-1,3] analytically. Justify your answers.

$f'(x) = 3x^2 - 12x$ $f'=0$ $f'=dne$ Endpts

Ex: Find all extrema of $f(x) = x^3 - 6x^2 + 1$ on [-1,3] analytically. Justify your answers.

$f'(x) = 3x^2 - 12x$

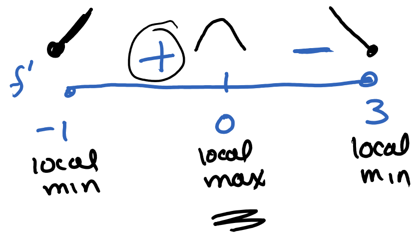
$0 = 3x^2 - 12x$

$= 3x(x-4)$

$f' = 0$: $x = 0, 4$

$f' = \text{dne}$: never

Endpts: $x = -1, 3$



x	$f(x)$
-1	-6
0	1 — Abs max
3	-26 — Abs min

f(x) has a local min of -6 @ x=-1, b/c x=-1 is an endpt and f' > 0 to the right of x = -1.

f(x) has an abs max of 1 @ x=0 b/c f' goes from + to − @ x=0

f(x) has an abs min of -26 @ x=3 b/c x=3 is an endpt and f' < 0 to the left of x=3

Try: Find all extrema of ______________________ on ______ analytically. Justify your answers.