

So You Want More Practice, eh?

- 1) Show that the function f satisfies the hypotheses of the Mean Value Theorem on the given interval. Find each value of c that satisfies the Mean Value Theorem.



If numbers are "bad"

$f(x) = 3x^2 + x - 5, [-1, 4]$

f is continuous on $[-1, 4]$ and differentiable on $(-1, 4)$.

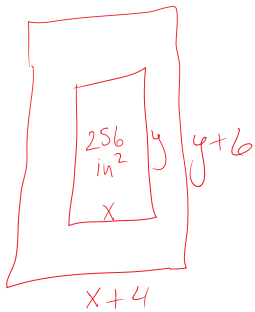
$\frac{f(4) - f(-1)}{4 - (-1)} = \frac{47 - (-3)}{5} = 10$

$6x + 1 = 10$

$6x = 9$

$x = \frac{9}{6} = \frac{3}{2}$

- 2) You are designing a rectangular poster to contain 256 in^2 of printing with a 3-in margin at the top and bottom and a 2-in margin at each side. What overall dimensions will minimize the amount of paper used? Round your answer to the nearest $\frac{1}{4}$ inch.



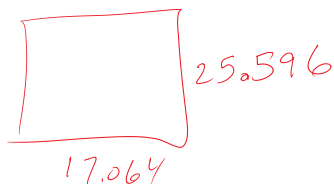
$A = (x+4)(y+6)$

$A = (x+4) \left(\frac{256}{x} + 6 \right)$

$A' = \frac{-}{13.064} \frac{+}{+}$

min dimensions are 17 in \times 25.5 in b/c when $x = 13.064$ A' changes from neg to pos

$xy = 256$
 $y = \frac{256}{x}$



- 3) If $f'(x) = (x+1)(x-2)^2$, then find the value of x at each point where f has a:

a) local maximum
None

$f' = \frac{-}{-1} \frac{+}{2} \frac{+}{+}$

- b) local minimum

$x = -1$

- c) point of inflection

$f''(x) = 1(x-2)^2 + 2(x-2)^1 \cdot (x+1)$
 $= (x-2)[(x-2) + 2(x+1)]$
 $= (x-2)(3x)$

$x = 2, x = 0$

$f'' = \frac{+}{0} \frac{+}{2}$

4) If the point (1, 6) is a point of inflection of the curve $y = x^3 + ax^2 + bx + 1$, find the values of a and b.

$$y = x^3 - 3x^2 + bx + 1$$

$$6 = 1^3 - 3(1)^2 + b(1) + 1$$

$$6 = 1 - 3 + b + 1$$

$$6 = -1 + b$$

$$7 = b$$

$$\boxed{a = -3}$$

$$\boxed{b = 7}$$

$$y' = 3x^2 + 2ax + b$$

$$y'' = 6x + 2a$$

$$0 = 6(1) + 2a$$

$$-6 = 2a$$

$$a = -3$$

$$x = 1$$

$$y'' = 0$$

5) If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true.

$$f' \quad \begin{array}{c} - \quad + \quad - \\ \hline -2 \quad \quad 2 \end{array}$$

(a) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.

(b) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.

(c) f has relative minima at $x = -2$ and $x = 2$.

(d) f has relative maxima at $x = -2$ and $x = 2$.

(e) It cannot be determined if f has any relative extrema.

6) Let $f(x) = x^3 - 3a^2x + 2a^3$ where a is a positive constant.

a) Find the intervals where f(x) is decreasing.

$$[-a, a] \text{ b/c } f' < 0$$

$$f'(x) = 3x^2 - 3a^2$$

$$3(x^2 - a^2) = 0$$

$$3(x+a)(x-a) = 0$$

$$x = -a, x = a$$

$$f' \quad \begin{array}{c} + \quad - \quad + \\ \hline -a \quad \quad a \end{array}$$

b) Find the relative maximum value of the function.

$$f(-a) = (-a)^3 - 3a^2(-a) + 2a^3$$

$$= -a^3 + 3a^3 + 2a^3$$

$$= 4a^3$$

$$\boxed{4a^3}$$

c) Find the point of inflection.

$$f''(x) = 6x$$

$$f'' \quad \begin{array}{c} - \quad + \\ \hline 0 \end{array}$$

$$x = 0$$

$$\boxed{(0, 2a^3)}$$