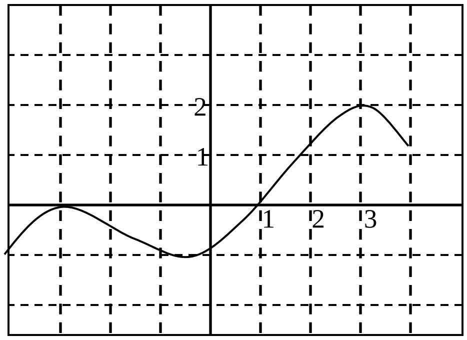
AP Calculus Name________________________________

Conceptual Derivative Practice

1. The graph of $f\,'$, the derivative of f, is shown below. Its domain is [-4, 4].



a. Suppose that $f\,(3) = 1$. Find an equation of the line tangent to f at the point (3, 1).

$y-1=2(x-3)$

b. Where does f have a local minimum? Justify your answer.

f has a local min @ $x=1$ since f' changed from − to + at $x=1$

c. Estimate $f\,''(2)$.

$f''(2) \approx 1$

d. Where does f have an inflection point? Justify your answer.

f has an inf. pt. at $x=-3$, $x=-1/2$, $x=3$ since f' changed from inc. to dec. OR dec to inc at these points.

e. Where does f achieve it maximum on the interval [1, 4]?

At $x=4$.

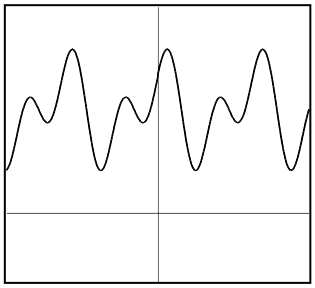
*2. If $f\,'(x) = \sin(\ln x)$, how many relative extrema does f have in the interval (0.5, 2]?

$\boxed{2}$

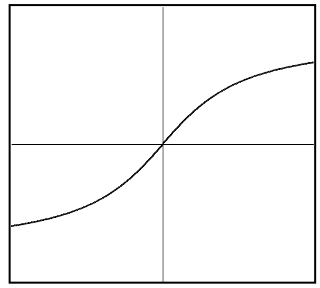
Mode

* means calculator acceptable if you choose to use it

3. The graphs of the <u>derivatives</u> of two functions f and g are given below.



$y = f'(x)$



$y = g'(x)$

a. How many solutions can the equation $f(x) = 0$ have? Explain.

Possibly 0 or 1 time since $f' > 0$.

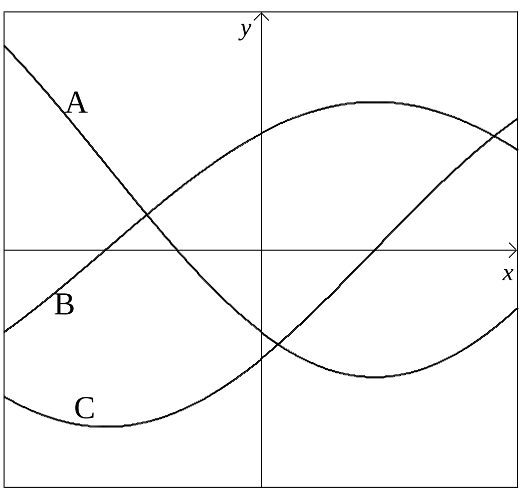
b. How many solutions can the equation $g(x) = 0$ have? Explain.

At most 2 times since g' went from − to + at $x = 0$.

c. If $g(x) = 0$ has two solutions, what can you say about where these two solutions lie? Justify your answer.

One must be when $x < 0$ and $x > 0$ since $g' < 0$ when $x < 0$ and $g' > 0$ when $x > 0$.

4. Graphs of f, f' and f'' appear below. Which is which? How can you tell?



$f \to A$

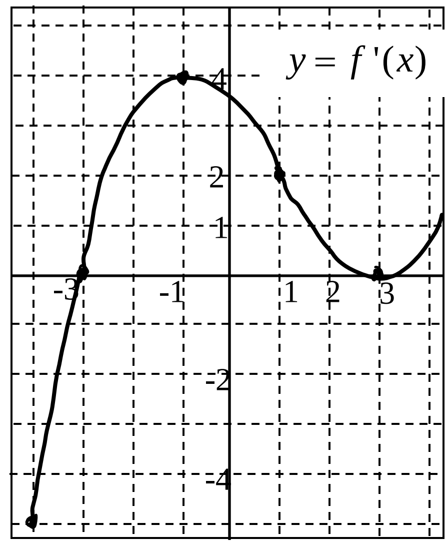
$f' \to C$

$f'' \to B$

*5. Two particles start at the origin and move along the x-axis. For $0 \le t \le 10$, their respective position functions are given by $x_1 = \sin(t)$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

a. none b. one c. two d. three e. four

6. Use the graph below to answer the following questions.



a. Suppose $f(1) = 5$. Find an equation of the line tangent to the graph of f at (1, 5).

$y - 5 = 2(x - 1)$

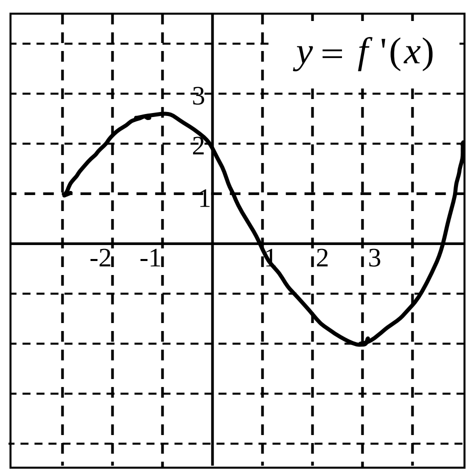
b. Suppose $f(-1) = -2$. Could $f(3) = -6$? Explain your answer.

No, since $f' > 0$ for $[-1, 3]$, then f is inc.

c. If $f(0) = 1$ order $f(0)$, $f'(0)$ and $f''(0)$ from least to greatest.

$f''(0) < f(0) < f'(0)$

7. The graph of f ',continuous and differentiable on the closed interval [-3, 5], is shown below. The graph has horizontal tangents at $x = -1$ and $x = 3$. Use the graph to answer the following questions.



a. What are the critical values of f?

$x = 1, x \approx 4.5$

b. Estimate the x-coordinate(s) of any local minima of f.

$x \approx 4.5$

c. On what interval is f both increasing and concave down?

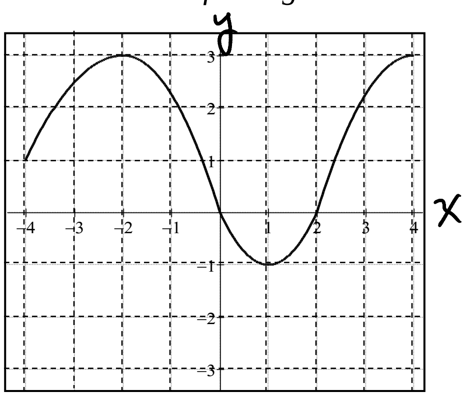
$f' > 0$ and f' dec.

$(-1, 1)$

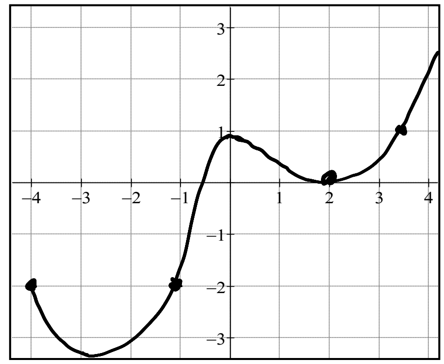
d. Let another function g be defined by $g(x) = x^2 - 3x - 1$. If $h(x) = f(g(x))$, find $h'(4)$.

$h'(4) = -10$

8. The functions f and g are defined on the interval [-4, 4] by the graphs below.



f



g

a. Estimate the rate of change of the function $f(g(x))$ at $x = 1$.

$\boxed{1/2}$

b. Is $f(g(x))$ increasing or decreasing at $x = 3$?

Decreasing

c. At what x-values does the graph of $y = f(g(x))$ have horizontal tangents?

$\boxed{x = -3,\ x = 0,\ x = 2,\ x = -1,\ x \approx 3.5}$

Conceptual Derivative Practice Answers

1a) $y = 2x - 5$

1b) $x = 1$; f' changes from – to + at 1

1c) Any $0.5 < m < 1.5$

1d) About $x = -3$, $x = -0.5$, and $x = 3$
These are places where f' has local extrema, so f'' goes from positive to 0 to negative or vice versa

1e) $x = 4$ since f' is nonnegative on [1, 4] so f increases on the interval

2) 2 relative extrema

3a) If the domain is limited to what we see, 0 or 1 solutions. F is continually rising and may or may not cross the x-axis

3b) g' indicates g is falling, then rising like a quadratic. g can have 0, 1 or 2 zeros.

3c) It appears they would be symmetric about the y-axis based on the symmetry of g'.

4) A is f, C is f' and B is f''

5) D

6a) $y = 2x + 3$

6b) No, f' is nonnegative on [-1, 3] so f is rising, not falling from –2 to –6

6c) $f''(0)$, $f(0)$, $f'(0)$

7a) $x = 1$, $x = 4.5$

7b) $x = -3$, $x = 4.5$

7c) (-1 , 1)

7d) -10

8a) $\frac{1}{4}$ to 1 (est)

8b) Decreasing

8c) $x = \{-4, -2.9, -1, 0, 2, 3.5\}$