

Finding Extrema & Inc/Dec Interval

Ex: for $f(x) = 3x^4 + 5$ on $[-2, 3]$,

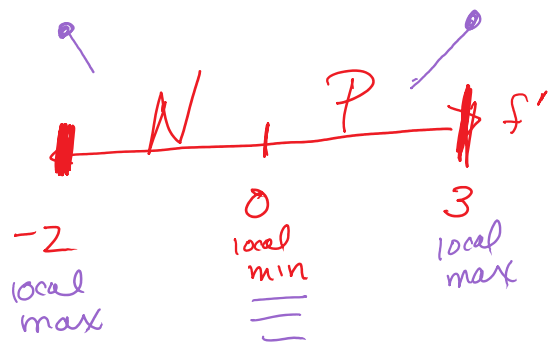
a. Find all extrema. Justify.

$$f'(x) = 12x^3$$

$$\frac{f' = 0}{12x^3 = 0}$$

$$x = 0$$

$$\frac{f' = \text{dne}}{\text{Never}}$$



x	f(x)
-2	53
0	5
3	248

$f(x)$ has a max of 248 @ $x = 3$
 b/c $x = 3$ is an endpt and $f' > 0$
 to the left of $x = 3$.

$f(x)$ has a local max of 53 @ $x = -2$,
 b/c $x = -2$ is an endpt and $f' < 0$
 to the right of $x = -2$.

$f(x)$ has a min of 5 @ $x = 0$
 b/c f' goes from $-$ to $+$ @
 $x = 0$.

b. where is f increasing? Justify

$f(x)$ is increasing $[0, 3]$, b/c $f' > 0$
 over the interval.

c. where is f decreasing? Justify

$f(x)$ is decreasing $[-2, 0]$, b/c $f' < 0$

over the interval.

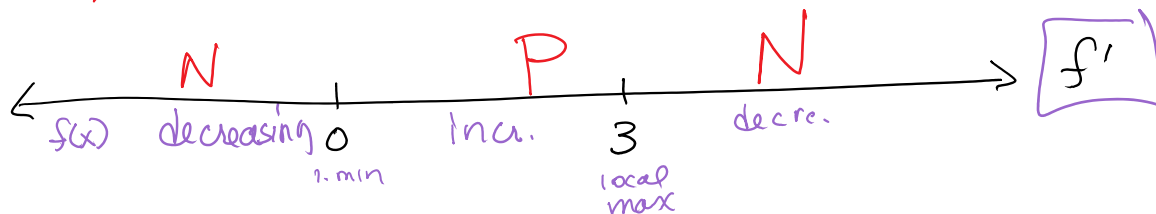
ex: given $f(x) = -\frac{2}{3}x^3 + 3x^2 + 5$

a. Find all local/relative extrema. Justify

$$f'(x) = -2x^2 + 6x$$

$$\begin{aligned} \underline{f' = 0} \\ -2x^2 + 6x &= 0 \\ -2x(x-3) &= 0 \\ x=0 \quad x=3 \end{aligned}$$

$$\underline{f' = \text{dne}} \\ \text{never}$$



$f(x)$ has a local min @ $x=0$, b/c f' goes from $-$ to $+$ @ $x=0$.

$f(x)$ has a local max @ $x=3$, b/c f' goes from $+$ to $-$ @ $x=3$.

b. When is $f(x)$ inc/dec. Justify.

$f(x)$ is decreasing from $(-\infty, 0] \cup [3, \infty)$, b/c f' is negative over the interval.

$f(x)$ is increasing from $[0, 3]$, b/c $f' > 0$ over the interval.

Find all critical points of $f(x) = xe^x$

$$f' = xe^x + e^x$$

$$e^x(x+1) = 0$$

$$x = -1 \quad \text{c.p.}$$