

Exponential and Log Derivatives

$$y = e^x \quad \boxed{y' = e^x} \quad \boxed{y = e^u \quad y' = e^u \cdot u'}$$

$$\boxed{y' = e^x \cdot \ln e}$$

example $y = e^{x^2} \quad \frac{dy}{dx} = e^{x^2} \cdot 2x$

$$y = e^{\sin x} \quad y' = e^{\sin x} \cdot \cos x$$

$$y = \ln x \quad \text{iff} \quad e^y = x$$

$$x = e^y$$

$$1 = e^y \cdot y' \quad y' = \frac{1}{e^y} = \frac{1}{x}$$

$$\boxed{\frac{d}{dx} \ln x = \frac{1}{x} \quad \frac{d}{dx} \ln u = \frac{1}{u} \cdot u'}$$

example: $f(x) = \ln(3x) \quad f'(x) = \frac{1}{3x} \cdot 3 = \frac{1}{x}$

Exponential $a = \text{constant}$
 $y = a^x$ $x = \text{is a variable/function}$

$$\ln y = \ln a^x$$

$$\ln y = x \cdot \ln a$$

constant

$$\star (y) \frac{1}{y} \cdot y' = \ln a \cdot y$$

$$y' = y \ln a$$

$$y' = a^x \ln a$$

$$\ln 2 \quad (\text{constant})$$

$$\frac{1}{\ln 2} \quad \text{constant}$$

$$y = 2 \cdot x \quad y' = 2$$

$$y' = 2 \cdot \frac{d}{dx} x$$

$$\frac{d}{dx} x \cdot \ln a$$

$$= \ln a(1) + x(0)$$

$$\frac{d}{dx} a^x = a^x \cdot \ln a \quad \frac{d}{dx} a^u = a^u \cdot \ln a \cdot u'$$

Ex:

$$y = 4^x$$

$$y' = 4^x \cdot \ln 4$$

or

$$y = 4^x$$

$$\ln y = \ln 4^x$$

$$\ln y = x(\ln 4)$$

$$\frac{1}{y} \cdot \frac{1}{y} \cdot y' = \ln 4 \cdot \frac{1}{y}$$

$$y' = y \ln 4$$

$$= 4^x \ln 4$$

you try... $y = 3^{\sin x}$

$$\frac{d}{dx} a^u = a^u \cdot \ln a \cdot u'$$

$$\frac{dy}{dx} = 3^{\sin x} \cdot \ln 3 \cdot \cos x$$

$$y' = 3^{\sin x} \ln 3 \cos x$$

$$y = \log_2 x$$

$$y = \frac{\ln x}{\ln 2} = \frac{1}{\ln 2} \cdot \ln x$$

↗ constant

$$y' = \frac{1}{\ln 2} \cdot \frac{d}{dx} \ln x$$

$$y' = \frac{1}{\ln 2} \cdot \frac{1}{x} = \frac{1}{x \ln 2}$$

change of base

$$\log_b a = \frac{\log_c a}{\log_c b} = \frac{\ln a}{\ln b}$$

fun precal fact