AP Calculus AB **Name:**
Cumulative Review – not comprehensive

Recall The Fundamental Theorem of Calculus:

If p is an antiderivative of f, then $\int_a^b f(t)dt = p(b) - p(a)$.

1. Let $g(x) = \int_{-7}^{8x^2+4} f(t)dt$. Find $g'(x)$.

$g(x) = F(8x^2+4) - F(-7)$

$g'(x) = F'(8x^2+4) \cdot 16x - 0$

$= 16x\, f(8x^2+4)$

2. Let $g(x) = \int_{\ln x}^{\cos 3x} f(t)dt$. Find $g'(x)$.

$g(x) = F(\cos 3x) - F(\ln x)$

$g'(x) = F'(\cos 3x) \cdot -\sin 3x \cdot 3 - F'(\ln x) \cdot \frac{1}{x}$

$= -3\sin 3x\, f(\cos 3x) - \frac{1}{x} f(\ln x)$

3. The function $g(x) = \frac{x^2}{e^x}$ has the derivative $g'(x) = \frac{x(2-x)}{e^x}$. Find the exact value of $\int_1^4 \frac{x(2-x)}{e^x}dx$.

$g(4) - g(1) = \frac{4^2}{e^4} - \frac{1^2}{e^1} = \frac{16}{e^4} - \frac{1}{e}$

4. If f is the function defined by $f(x) = \sqrt{\cos\left(\frac{1}{5}x\right)}$ and g is an antiderivative of f such that $g(2) = 5$, then use a calculator to approximate $g(6)$.

$\int_2^6 \sqrt{\cos(\frac{1}{5}x)}\, dx = 3.2685 = g(6) - g(2)$

$g(6) = 8.2685$

5. If f is the function defined by $f(x) = \frac{1}{5x^2+3}$ and g is an antiderivative of f such that $g(3) = 11$, then use a calculator to approximate $g(1)$.

$\int_1^3 \frac{1}{5x^2+3}\, dx = 0.1049 = g(3) - g(1)$

$g(1) = 10.8951$

6. Let f and h be twice differentiable functions.

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	0	3	4	-2	8	1	0	4	1	7
$f'(x)$	-2	-3	-4	-5	-6	2	-2	3	23	-2
$h(x)$	1	2	1	4	10	5	-4	2	3	4
$h'(x)$	5	4	3	2	1	6	-6	1	4	8

a. Evaluate $\int_1^3 h'(x)dx$.

$h(3) - h(1) = 4 - 2 = 2$

b. Let $a(x) = f(2x)$.

i. What is $a'(x)$?

$a'(x) = f'(2x) \cdot 2$

ii. Evaluate $\int_1^3 a'(x)dx$.

$a(3) - a(1) = f(6) - f(2) = 0 - 4 = -4$

iii. Evaluate $\int_1^3 f'(2x)dx$

$\frac{1}{2} \cdot -4 = -2$

c. Let $b(x) = f(h(x))$.

i. What is $b'(x)$?

$b'(x) = f'(h(x)) \cdot h'(x)$

ii. Evaluate $\int_1^3 b'(x)dx$.

$b(3) - b(1) = f(h(3)) - f(h(1)) = f(4) - f(2) = 8 - 4 = 4$

iii. Evaluate $\int_2^5 f'(h(x)) \cdot h'(x)dx$

$b(5) - b(2) = f(h(5)) - f(h(2)) = f(5) - f(1) = 1 - 3 = -2$

d. Let $m(x) = \int_{-3}^{e^{2x}} f(t)dt$.

i. What is $m'(x)$?

$m'(x) = f(e^{2x}) \cdot 2e^{2x}$

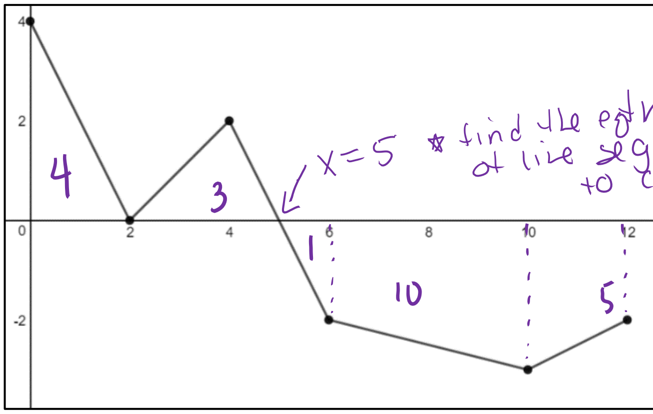
ii. Find $m'(0)$.

$m'(0) = f(1) \cdot 2$

$= 3 \cdot 2$

$= 6$

7. The graph below is *f(x)*. Let $g(x)$ be an antiderivative of *f* over the interval of $0,12$ and $g\ 5 = 2$.



a. Calculate *g(0)*, *g(2)*, and *g(10)*.

$g(0) = -5$

$g(2) = -1$

$g(10) = -9$

$\int_5^x f(t)\,dt = g(x) - g(5)$

$g(5) + \int_5^x f(t)\,dt = g(x)$

$g(x) = 2 + \int_5^x f(t)\,dt$

b. Make a sign chart for *g'(x)* and *g''(x)*.

$g' = f$: 0 (+) 2 (+) 5 (−) 12

$g'' = f'$: 0 (−) 2 (+) 4 (−) 6 (−) 10 (+) 12

c. Where is *g(x)* increasing? Justify your response.

$[0,5]$ because $g' = f > 0$.

d. Where is *g(x)* decreasing? Justify your response.

$[5,12]$ because $g' = f < 0$

e. Where is *g(x)* concave up? Justify your response.

$(2,4) \cup (10,12)$ because $g'' = f' > 0$.

f. Where is *g(x)* concave down? Justify your response.

$(0,2) \cup (4,10)$ because $g'' = f' < 0$.

g. Where does *g(x)* have points of inflection? Justify your response.

$x = 2, 4, 10$ because $g'' = f'$ changes sign.

h. Where does *g(x)* have local minima? Justify your response.

$x = 0$ because $g' = f > 0$ on $(0,5)$.

$x = 12$ because $g' = f < 0$ on $(5,12)$.

i. Where does *g(x)* have local maxima? Justify your response.

$x = 5$ because $g' = f$ changes from (+) to (−).

j. What is the minimum value of *g(x)*? Justify your response.

$g(0) = -5$ $g(12) = -14$ ∴ The minimum value is -14 because $g(12) < g(0)$.

k. What is the maximum value of *g(x)*?

$g(5) = 0$

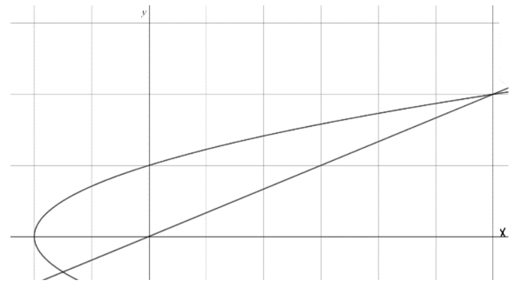
8. Find the area between the lines and curves of $x - y^2 = -4$ *and* $y = \frac{1}{3}x$.

$x = y^2 - 4 \qquad x = 3y$

$3y = y^2 - 4 \qquad y^2 - 3y - 4$

$(y-4)(y+1) = 0$

$y = 4 \quad y = -1$



$$\text{Area} = \int_{-1}^{4} \left[3y - (y^2 - 4)\right] dy = \int_{-1}^{4} (3y - y^2 + 4)\, dy = 125/6$$

9. Show an integral set up (do not solve) the gives the area enclosed in the first quadrant from $(0,2)$ between $y = 2\sqrt{x},\ y = (x-2)^2 + 1,$ *and* $y = \frac{1}{2}x$.

$$A = \int_{0}^{1} \left(2\sqrt{x} - \tfrac{1}{2}x\right) dx + \int_{1}^{2} \left[\left((x-2)^2 + 1\right) - \tfrac{1}{2}x\right] dx$$

10. (Calc ok) Find the area between the curves $f(x) = 3\sqrt[3]{x}$ *and* $g(x) = x^3$. You must show your integral set up.

$a \approx -1.510 \qquad b \approx 1.510$

$$\text{Area} = \int_{a}^{0} \left(x^3 - 3\sqrt[3]{x}\right) dx + \int_{0}^{b} \left(3\sqrt[3]{x} - x^3\right) dx$$

or

$$2\int_{a}^{0} \left(x^3 - 3\sqrt[3]{x}\right) dx \approx 5.196$$