4.4 Day 1

Tuesday, September 24, 2019 8:16 AM

Derivatives of Exponential & Log Functions

u= is a function of x

Graph $y = e^x$

$\frac{d}{dx} e^x \Big|_{x=x} = e^x$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} e^u = e^u \cdot u'$

$f(g(x))$

$f(x) = e^x$

$g(x) = \sec x$

$f(g(x)) = e^{\sec x}$

$f'(g(x)) \cdot g'(x) = e^{\sec x} \cdot \sec x \tan x$

Example: Find $\frac{dy}{dx}$ of $y = e^{3x^4 - x}$

$\frac{dy}{dx} = e^{3x^4 - x} \cdot (12x^3 - 1)$

you try... $\frac{d}{dx} e^{\sec x} = e^{\sec x} \cdot \sec x \tan x$

$y = 2^x$

$\frac{d}{dx} 2^x \Big|_{x=x}$

$y = 2^x$

$\ln y = x \cdot \ln 2$

$\frac{1}{y} \cdot y' = \ln 2$

$y' = y \cdot \ln 2$

$y' = 2^x \cdot \ln 2$

$y = \ln x$

$e^y = x$

$e^y \cdot y' = 1$

$y' = \frac{1}{e^y} = \frac{1}{x}$

$\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} \ln u = \frac{1}{u} \cdot u'$

$\frac{d}{dx} a^x = a^x \cdot \ln a$ (ln a = constant)

where a = constant $a > 0$ $a \neq 1$

constant

$$\frac{d}{dx} a^u = a^u \cdot \ln a \cdot u'$$

Examples

a. find y' given $y = \ln(3x^2 - 5x)$

$$y' = \frac{1}{3x^2-5x} \cdot (6x-5) = \frac{6x-5}{3x^2-5x}$$

you try... $\frac{d}{dx} \ln\left(\frac{1}{x}\right)$ $x \neq 0$ $x > 0$

$$= \frac{1}{\left(\frac{1}{x}\right)} \cdot (-x^{-2})$$

$$= x \cdot (-x^{-2})$$

$$= -x^{-1} = -\frac{1}{x}$$

you try... $\frac{d}{dx} 3^{\ln(x^2)} = 3^{\ln(x^2)} \cdot \ln 3 \cdot \frac{1}{x^2} \cdot 2x$

$$= 3^{\ln(x^2)} \cdot \frac{2 \cdot \ln 3}{x}$$

Let $y = \log_2 x$ find y'

$$y = \frac{\ln x}{\ln 2}$$

$$y = \frac{1}{\ln 2} \cdot \ln x$$

$$y' = \frac{1}{x} \cdot \frac{1}{\ln} - \frac{1}{x \ln 2}$$

$y' = \frac{1}{\ln 2} \frac{d}{dx} \ln x$

$a > 0$
$a \neq 1$

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a} \qquad \frac{d}{dx} \log_a u = \frac{1}{u \ln a} \cdot u'$$

$$\frac{d}{dx} \log_2 (x^2) = \frac{1}{x^2 \ln 2} \cdot 2x = \frac{2}{x \ln 2}$$