4.3 Day 2 (Monday 9/23)

Friday, September 20, 2019 10:01 AM

AP Calculus BC 4.3 Notes Day 2 Name________________________

Derivatives of Inverse Functions

Find each of the following derivatives at the given x-values.

1. $f(x)=x^2$ at $x=3$ $(3,9)$

$f'(x)=2x\Big|_{x=3}=6$

$g(x)=\sqrt{x}$ at $x=9$ $(9,3)$ $\Rightarrow$ $g'(9)=\dfrac{1}{f'(3)}$

$g'(x)=\frac{1}{2}x^{-1/2}\Big|_{x=9}=\frac{1}{6}$

2. $f(x)=x^3$ at $x=2$ $(2,8)$

$f'(x)=3x^2\Big|_{x=2}=12$

$g(x)=\sqrt[3]{x}$ at $x=8$ $(8,2)$ $g'(8)=\dfrac{1}{f'(2)}$

$g'(x)=\frac{1}{3}x^{-2/3}\Big|_{x=8}=1/12$

$f'(2)=\dfrac{1}{g'(16)}$

3. $f(x)=x^4$ at $x=2$

$f'(x)=4x^3\Big|_{x=2}=32$

$g(x)=\sqrt[4]{x}$ at $x=16$

$g'(x)=\frac{1}{4}x^{-3/4}\Big|_{x=16}=1/32$

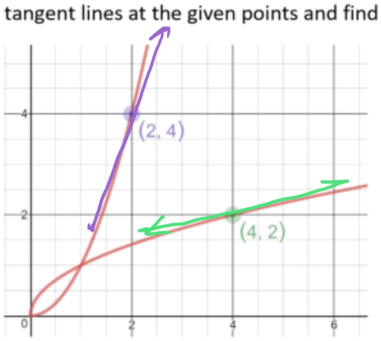
4. $f(x)=x^5$ at $x=2$

$f'(x)=5x^4\Big|_{x=2}=80$

$g(x)=\sqrt[5]{x}$ at $x=32$

$g'(x)=\frac{1}{5}x^{-4/5}\Big|_{x=32}=\frac{1}{80}$

5. Below are the graphs of two functions $f(x)=x^2$ and its inverse $f^{-1}(x)=\sqrt{x}$. Label the functions, draw in the tangent lines at the given points and find the slopes of those tangent line at those points.



$f(x) \Rightarrow$ p.o.t $(2,4)$

$m=f'(2)=4$

$f'(x)=2x$

$f'(2)=4$

$y-4=4(x-2)$

$f^{-1}(x) \Rightarrow$ p.o.t $(4,2)$

$m=(f^{-1})'(4)=\frac{1}{4}$

$(f^{-1})'(x)=\frac{1}{2}x^{-1/2}$

$(f^{-1})'(4)=\frac{1}{4}$

$y-2=\frac{1}{4}(x-4)$

6. Below are the graphs of two functions $f(x)=x^3$ and its inverse $f^{-1}(x)=\sqrt[3]{x}$. Label the functions, draw in the tangent lines at the given points and find the slopes of those tangent lines at those points.

$f(x) \Rightarrow$ p.o.t $(2,8)$ $\quad f^{-1}(x)$ pot $\Rightarrow (8,2)$

$f'(x)=m=12$ $\quad (f^{-1})'(x)=m=\frac{1}{12}$

$y-8=12(x-2)$ $\quad y-2=\frac{1}{12}(x-8)$

How are $f(x)$ and $g(x)$ in each problem(#1-6) related to each other? How are the derivative values in each problem above related to each other?

1. $f(x)$ & $g(x)$ inverse function
2. the derivatives/slopes of tangent line are reciprocals

Based on your observations, write a rule relating the derivative of a function and its inverse.

$f(x)=(a,b)$ $\quad f^{-1}(x)=(b,a)$

$\boxed{(a, f(a))}$ $\quad (b, f(b))$

$$\frac{d}{dx}f(x)\Big|_{x=a}=\frac{1}{\frac{d}{dx}f^{-1}(x)}\Bigg|_{x=f(a)}$$

$$\frac{d}{dx}f^{-1}(x)\Big|_{x=f(a)}=\frac{1}{\frac{d}{dx}f(x)}\Bigg|_{x=a}$$

Why does this work??? Here's a proof of what we just came up with.

$$f(g(x)) = x$$

$$f'(g(x)) \cdot g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$g'(x) = \frac{1}{f'(y)}$$

side note

$f(g(x)) = x$ $\underbrace{g(f(x)) = x}_{\text{inverses}}$

$f(x) = y$

$g(x) = y$

Example from AP Exam:

Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $\boxed{f'(6) = -2}$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

$$\boxed{\begin{aligned} g(x) &= (3, 6) \\ f(x) &= (6, 3) \end{aligned}}$$

A) $-\frac{1}{2}$

$$g'(3) = \frac{1}{f'(6)} = -\frac{1}{2}$$

B) $-\frac{1}{8}$

C) $\frac{1}{6}$

D) $\frac{1}{3}$

E) The value of $g'(3)$ cannot be determined from the information given.