4.3 Day 2

Friday, October 13, 2017 8:01 AM

AP Calculus AB 4.3 Notes Day 2 Name______________________________

Derivatives of Inverse Functions

Find each of the following derivatives at the given x-values.

1. $f(x)=x^2$ at $x=3$ $(3,9)$ $g(x)=\sqrt{x}$ at $x=9$ $(9,3)$

$f'(x)=2x\Big|_{x=3}=6$ $g'(x)=\frac{1}{2}x^{-1/2}\Big|_{x=9}=\frac{1}{6}$

2. $f(x)=x^3$ at $x=2$ $(2,8)$ $g(x)=\sqrt[3]{x}$ at $x=8$ $(8,2)$

$f'(x)=3x^2\Big|_{x=2}=12$ $g'(x)=\frac{1}{3}x^{-2/3}\Big|_{x=8}=\frac{1}{12}$

3. $f(x)=x^4$ at $x=2$ $(2,16)$ $g(x)=\sqrt[4]{x}$ at $x=16$ $(16,2)$

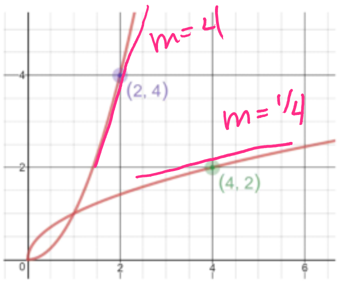
$f'(x)=4x^3\Big|_{x=2}=32$ $f'(2)=\frac{1}{g'(16)}=\frac{1}{(f^{-1})'(16)}$ $g'(x)=\frac{1}{4}x^{-3/4}\Big|_{x=16}=\frac{1}{32}$

4. $f(x)=x^5$ at $x=2$ $(2,32)$ $g(x)=\sqrt[5]{x}$ at $x=32$ $(32,2)$

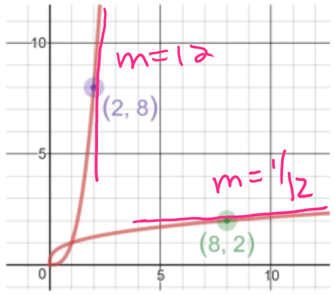
$f'(x)=5x^{-4/5}\Big|_{x=2}=80$ $g'(x)=\frac{1}{5}x^{-4/5}\Big|_{x=32}=\frac{1}{80}$

$g'(32)=(f^{-1})'(32)=\frac{1}{f'(2)}$

5. Below are the graphs of two functions $f(x)=x^2$ and its inverse $f^{-1}(x)=\sqrt{x}$. Label the functions, draw in the tangent lines at the given points and find the slopes of those tangent line at those points.



6. Below are the graphs of two functions $f(x) = x^3$ and its inverse $f^{-1}(x) = \sqrt[3]{x}$. Label the functions, draw in the tangent lines at the given points and find the slopes of those tangent lines at those points.



$f'(x) = 3x^2 \Big|_{x=2} = 12$

$x =$

How are $f(x)\, and\, g(x)$ in each problem(#1-6) related to each other? How are the derivative values in each problem above related to each other?

1. $f(x)$ & $g(x)$ are inverses

2. Reciprocal relationship

Based on your observations, write a rule relating the derivative of a function and its inverse.

$f(x) = (a, b) \qquad f^{-1}(x) = (b, a)$

$$(f^{-1})'(b) = \frac{1}{f'(a)}$$

$$(f^{-1})'(a) = \frac{1}{f'(b)}$$

Why does this work??? Here's a proof of what we just came up with.

If $f(g(x)) = x$ & $g(f(x)) = x$

then $f(x)$ & $g(x)$ are inverses

$\therefore g(x) = f^{-1}(x)$

Find $g'(x)$

$f(g(x)) = x$

$f'(g(x)) \cdot g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{f'(y)}$

$g'(x) = (f^{-1})'(x) = \frac{1}{f'(y)}$

Example from AP Exam:

Let f be a differentiable function such that ~~$f(3) = 15$~~, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

$g'(3) = (f^{-1})'(3)$

$f^{-1}(x) = (3, 6)$

$f(x) = (6, 3)$

A) $-\frac{1}{2}$

B) $-\frac{1}{8}$

C) $\frac{1}{6}$

D) $\frac{1}{3}$

E) The value of $g'(3)$ cannot be determined from the information given.

$g'(3) = (f^{-1})'(3) = \frac{1}{f'(6)}$

$= \frac{1}{-2}$

$f(g(x)) = x$

$f'(g(x)) \cdot g'(x) = 1$

$\frac{1}{f'(g(x))} = \frac{1}{f'(y)}$

$f(x) = x^4 - 2x^3 + x + 6$

$f(1) = 1 - 2 + 1 + 6 = 6$

$f(x) = (1, 6)$

$f'(x) = 4x^3 - 6x^2 + 1$

find $(f^{-1})'(6)$

a. $f(1) = 6$

b. $(f^{-1})'(6) = \frac{1}{f'(1)}$

$= \frac{1}{-1} = -1$

$f'(x) = 4x^3 - 6x^2 + 1$

$f'(1) = 4 - 6 + 1 = -1$

$= \frac{1}{-1} = -1$

$(f^{-1})'(6) = -1$