

Derivatives of Inverse Trig. Functions

Find $\frac{d}{dx}(\sin^{-1}x)$

$$y = \sin^{-1}x$$

$$\sin y = x$$

$$\cos y \cdot y' = 1$$

$$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$

+ think about $\cos^2 y + \sin^2 y = 1$

$$\cos^2 y = 1 - \sin^2 y$$

$$\cos y = \sqrt{1 - \sin^2 y}$$

$$\cos y = \sqrt{1-x^2}$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \quad |x| < 1$$

$$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \quad |x| < 1$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x| \cdot \sqrt{x^2-1}} \quad |x| > 1$$

$$\frac{d}{dx} \csc^{-1} x = \frac{-1}{|x| \sqrt{x^2-1}} \quad |x| > 1$$

Ex! Find $\frac{dy}{dx}$ of $y = \cos^{-1}(4x^3)$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(4x^3)^2}} \cdot 12x^2 = \frac{-12x^2}{\sqrt{1-16x^6}}$$

Ex Find f' given $f(x) = \tan^{-1}\left(\frac{x}{2}\right)$

$$f'(x) = \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1+\frac{x^2}{4}\right)}$$

$$= \frac{1}{2 + \frac{x^2}{2}} \cdot \frac{(2)}{(2)} = \frac{2}{4 + x^2}$$

you try Differentiate!

a. $y = \csc^{-1}(\sqrt{x})$

$$y' = \frac{-1}{|\sqrt{x}| \sqrt{x-1}} \cdot \frac{1}{2} x^{-1/2}$$

$$= \frac{-1}{2|\sqrt{x}| \sqrt{x-1}}$$

$$= \frac{-1}{2|x| \sqrt{x-1}}$$

b. $y = \sin^{-1}\left(\frac{1}{x^3}\right)$

$$y' = \frac{1}{\sqrt{1 - \left(\frac{1}{x^3}\right)^2}} \cdot (-3x^{-4})$$

$$= \frac{1}{\sqrt{1 - \frac{1}{x^6}}} \cdot \frac{-3}{x^4}$$

$$= \frac{-3}{x^4 \sqrt{1 - \frac{1}{x^6}}} = \frac{-3}{x^4 \sqrt{\frac{x^6}{x^6} - \frac{1}{x^6}}}$$

$$= \frac{-3}{x^4 \sqrt{\frac{x^6 - 1}{x^6}}}$$

$$= \frac{-3}{\frac{x^4 \sqrt{x^6 - 1}}{\sqrt{x^6}}} = \frac{-3}{\frac{x^4}{x^3} \cdot \sqrt{x^6 - 1}}$$

$$= \frac{-3}{x \sqrt{x^6 - 1}}$$