4.3 Day

Tuesday, October 11, 20[illegible] 12:48 PM

Derivatives of Inverse Trig.

$\frac{d}{dx}\sin^{-1}x$

Think about $y=\sin x$

$\boxed{x=\sin y}$ $(\Rightarrow \sin^{-1}x)$

$1=\cos y\cdot y'$

$y'=\frac{1}{\cos y}$

$y'=\frac{1}{\sqrt{1-\sin^2 y}}$

$=\frac{1}{\sqrt{1-(\sin y)^2}}$ ☆

$y'=\frac{1}{\sqrt{1-x^2}}$

☆ $\sin^2 y+\cos^2 y=1$

$\cos^2 y=1-\sin^2 y$

$\cos y=\sqrt{1-\sin^2 y}$

Chain Rule

☆ $\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}\sin^{-1}u=\frac{1}{\sqrt{1-u^2}}\cdot u'$ ☆

ex: $\frac{d}{dx}\sin^{-1}(2x)=\frac{1}{\sqrt{1-(2x)^2}}\cdot 2=\frac{2}{\sqrt{1-4x^2}}$

☆ $\frac{d}{dx}\cos^{-1}x=\frac{-1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}\cos^{-1}u=\frac{-1}{\sqrt{1-u^2}}\cdot u'$

☆ $\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$ $\qquad$ $\frac{d}{dx}\tan^{-1}u=\frac{1}{1+u^2}\cdot u'$

$\frac{d}{dx}\cot^{-1}x=\frac{-1}{1+x^2}$ $\qquad$ $\frac{d}{dx}\cot^{-1}u=\frac{-1}{1+u^2}\cdot u'$

☆ $\frac{d}{dx}\sec^{-1}x=\frac{1}{|x|\sqrt{x^2-1}}$ $\qquad$ $\frac{d}{dx}\sec^{-1}u=\frac{1}{|u|\sqrt{u^2+1}}u'$

☆ $\frac{d}{dx}\csc^{-1}x=\frac{-1}{|x|\sqrt{x^2-1}}$ $\qquad$ $\frac{d}{dx}\csc^{-1}u=\frac{-1}{|u|\sqrt{u^2+1}}u'$

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examples: find y'

a. $y=\tan^{-1}(x^2-1)$

$y'=\frac{1}{1+(x^2-1)^2}(2x)$

b. $y=\csc^{-1}(\cos x)$

$y'=\frac{-1}{|\cos x|\sqrt{\cos^2 x-1}}\cdot(-\sin x)$

c. $y=\sin(x^3-2x)$

$y'=\cos(x^3-2x)(3x^2-2)$

d. $y=\cos^{-1}(\sqrt{x})$

$y'=\frac{-1}{\sqrt{1-x}}\cdot\frac{1}{2}x^{-1/2}$

e. $y=\sin(\cos^{-1}x)$

$y'=\cos(\cos^{-1}x)\cdot\frac{-1}{\sqrt{1-x^2}}$

$=$

f. $y=\sec^{-1}(3x)$

$y'=\frac{1}{|3x|\sqrt{9x^2-1}}\cdot 3$