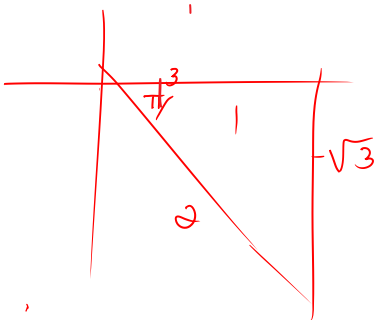
Precalculus Honors
Study Guide 4.3-4.5 and 4.7

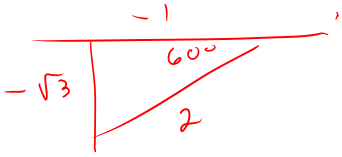
Non-Calculator Review Questions:



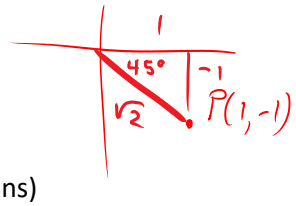
1. Evaluate each expression. Leave in exact form:

   a) $\cot\left(\frac{5\pi}{3}\right)$ $-\frac{1}{\sqrt{3}}$

   b) Sin 240° $-\frac{\sqrt{3}}{2}$



2. Point P(1, -1) is on the terminal side of angle θ.



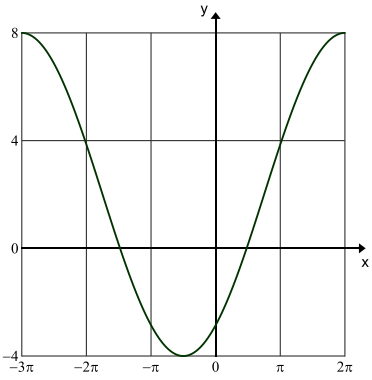
   a) Evaluate $\sec\theta$ $\sqrt{2}$

   b) Give the smallest positive measure for θ (give your answer in radians)

$-45^\circ + 360^\circ = 315^\circ$

Write an equation for the graph shown

$y = -3\sin\left[2\left(x - \frac{\pi}{4}\right)\right] + 5$



any or equivalent

$y = 2 + 6\cos\left[\frac{2}{5}(x + 3\pi)\right]$ -2π

$y = 2 - 6\cos\left[\frac{2}{5}\left(x + \frac{\pi}{2}\right)\right]$

$y = 2 + 6\sin\left[\frac{2}{5}\left(x - \frac{3\pi}{4}\right)\right]$

$y = 2 - 6\sin\left[\frac{2}{5}\left(x + \frac{7\pi}{4}\right)\right]$

3. The town of Monotony has a very odd weather pattern: every day's temperatures follow the same pattern, with a high temperature of 80° at 2:00 pm, and a low temperature of 58° at 2:00 am. Let t represent the number of hours since midnight (on some given day) and write a sinusoidal model for the temperature as a function of t.

Period = 24

$B = \frac{2\pi}{24} = \frac{\pi}{12}$

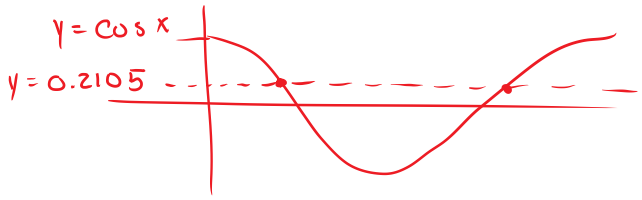
$C = \frac{80+58}{2} = \frac{138}{2} = 69^\circ$

$D = 14$ (2pm)

$A = \frac{80-58}{2} = \frac{22}{2} = 11^\circ$

$y = 69 + 11\cos\left[\frac{\pi}{12}(t - 14)\right]$

Calculator Review Questions:



Solve $\sec x = 4.75$ for $0 \le x \le 2\pi$

$\cos x = \frac{1}{4.75} \approx 0.2105$

$x \in \{1.359, 4.925\}$

3. List 3 angles that are coterminal with $-\frac{2\pi}{3}$ (do not include $-\frac{2\pi}{3}$)

any 3 of : ... $\frac{-14\pi}{3}$, $-\frac{8\pi}{3}$, *, $\frac{4\pi}{3}$, $\frac{10\pi}{3}$, $\frac{16\pi}{3}$, ...

4. Identify the asymptotes of $y = 2 + \cot\left(\frac{x}{2}\right)$

$y = \cot x \rightarrow$ asymptotes at $x = k\pi$

horiz stretch b.a.f.o. 2 $\rightarrow$ $x = 2k\pi$ for any integer k

..., $x = -4\pi$, $x = -2\pi$, $x = 0$, $x = 2\pi$, ...

5. Let $f(x) = 3 - 2\sec(4x)$

   a. Identify the period of $f(x)$

   $\frac{2\pi}{4} = \frac{\pi}{2}$

   b. Identify the domain of $f(x)$

   $\cos(4x) = 0$ when $4x =$ odd mult of $\frac{\pi}{2}$ $\rightarrow$ $x \neq$ odd mult of $\frac{\pi}{8}$

   $x \neq -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \ldots$

   c. Identify the range of $f(x)$

$3 - 2 = 1$

$3 + 2 = 5$

$y \le 1$ or $y \ge 5$

Part I: Sketch a graph of the following without using your calculator. Graph in radians and graph 2 periods! Make sure you label scales on BOTH axes. Plot critical values as well.

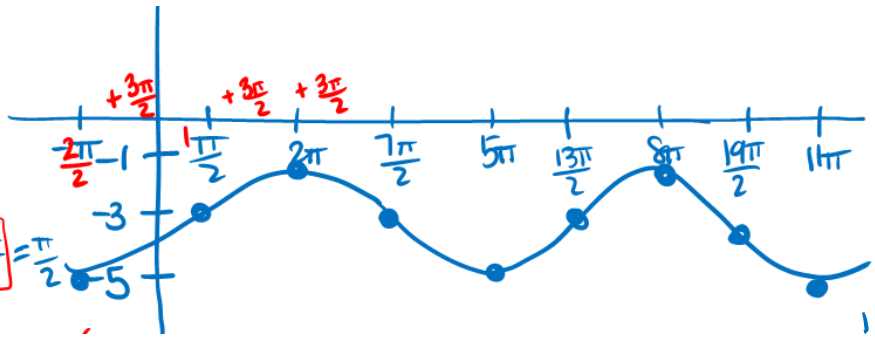
$= \frac{1}{2}\sin 2\left(x - \frac{\pi}{8}\right) + 1$

1) $y = -2\cos\frac{1}{3}(x+\pi) - 3$

Per $= \frac{2\pi}{\frac{1}{3}} = 6\pi$

$D = -\pi$

Crit pts every $\frac{6\pi}{4} = \frac{3\pi}{2}$ $\quad -\frac{2\pi}{2} + \frac{3\pi}{2} = \frac{\pi}{2}$



2) $y = \frac{1}{2}\sin\left(2x - \frac{\pi}{4}\right) + 1$ $= \frac{1}{2}\sin 2\left(x - \frac{\pi}{8}\right) + 1$

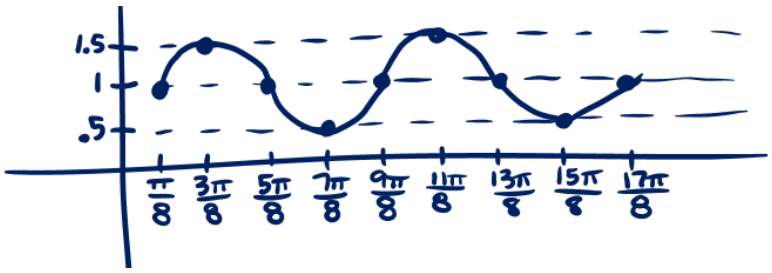
$A = \frac{1}{2}$

$C = 1$ (up 1)

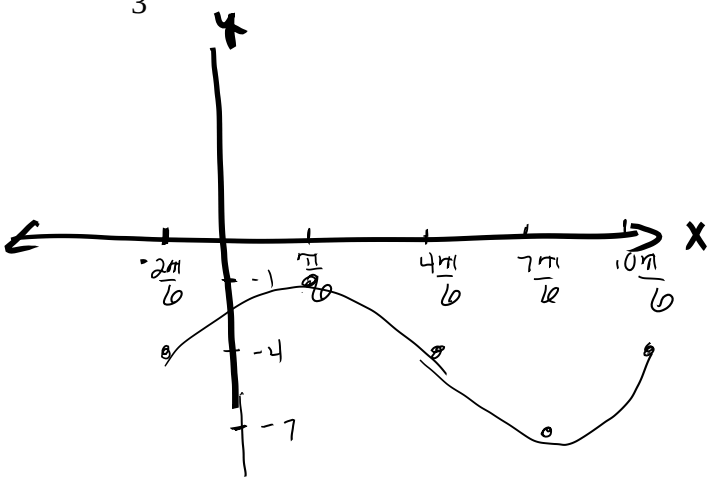
$B = 2$ Per $= \frac{2\pi}{2} = \pi$

Crit pts every $\frac{\pi}{4} = \frac{2\pi}{8}$

$D = \frac{\pi}{8}$

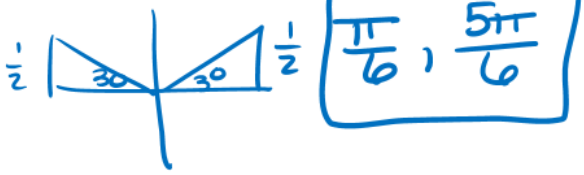


3) $y = 3\sin(x + \frac{\pi}{3}) - 4$

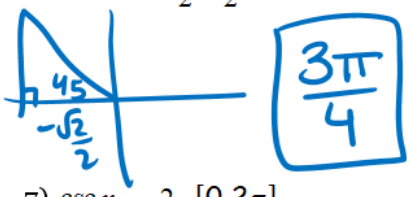


Part II: Without using your calculator, solve the equations over the given interval. Express your answers in radians.

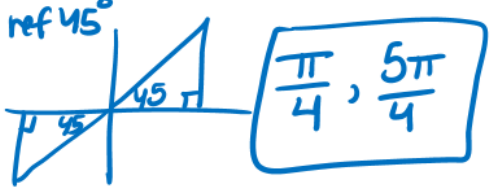
4) $\sin x = \frac{1}{2}$, $[0, 2\pi]$



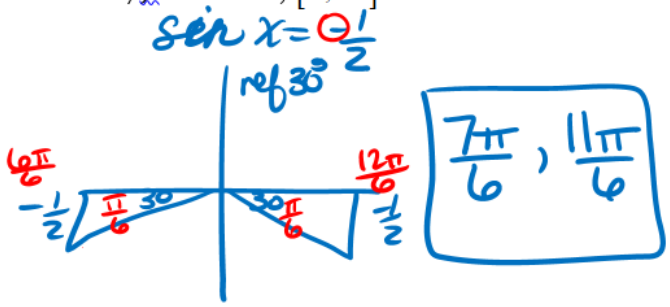
5) $\cos x = -\frac{\sqrt{2}}{2}; \frac{\pi}{2} \le x \le \pi$



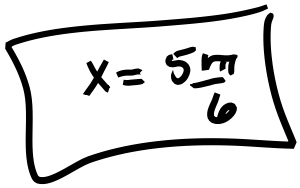
6) $\tan x = 1; 0 \le x \le 2\pi$



7) $\csc x = -2$, $[0, 2\pi]$



8) $\cot x = -\sqrt{3}; \frac{\pi}{2} \le x \le \pi$

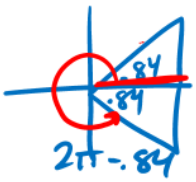


Part III: Use a calculator to solve the following equations over the given interval.

9) $\sin x = \frac{3}{4}; 0 \le x \le \frac{\pi}{2}$

$x = .85$

10) $\cos x = \frac{2}{3}; 0 \le x \le 2\pi$



$x = .84, 5.44$

11) $\csc x = -1.5; \pi \le x \le \frac{3\pi}{2}$

$\theta = \sin^{-1}\left(\frac{1}{1.5}\right)$

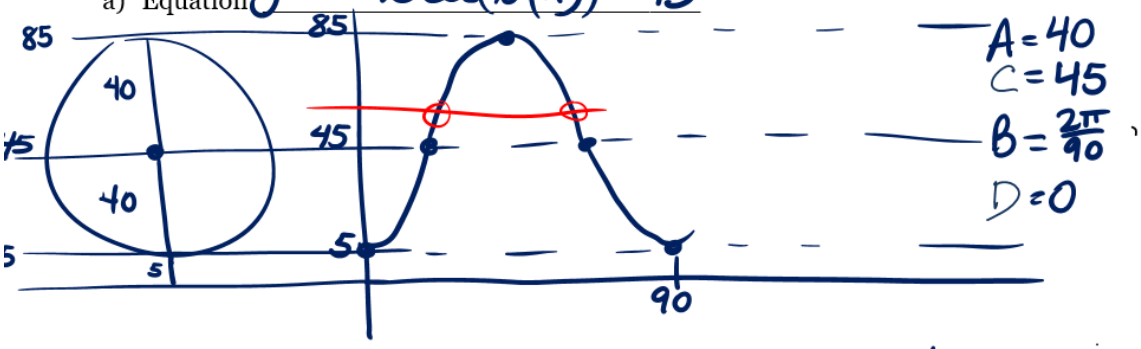
$\theta = .73$

$x = \pi + .73 = 3.87$

Part IV: Write a sinusoidal equation to describe the following situations. Then, use your equation to answer the accompanying questions.

12) A Ferris wheel is 80 feet in diameter and its center is 45 feet above the ground. It takes the Ferris wheel 1.5 minutes to complete a revolution. Per = 1.5 min = 90 sec

a) Equation: $y = -40\cos\left(\frac{2\pi}{90}(x)\right) + 45$



b) After how many ***seconds*** will the rider be at a height of 50 feet? 24.3 sec, 66 sec

$y_2 = 50$ Intersect

c) How high will the rider be after 1 **minute** into the ride? 65 ft

VALUE $X = 60$

13) The temperature in our classroom varies sinusoidally during the school day. At 8am, the room is at a high temperature of 75 degrees (F). Your friend records the temperature at 9:30 to be at a low of 62 degrees. (Note, the day ***really*** starts at midnight or 12 am. Be sure to adjust your graph and solutions accordingly!)

a) Equation: $y = 68.5 + 6.5\cos\left[\frac{2\pi}{3}(x - 8)\right]$

b) What will the temperature be at 3:00pm? 65.25 degrees

c) When is the ***first*** time the temperature will reach its low value of 62 degrees?

12:30 AM

Part V: Sketch 2 periods of each of the following functions. Plot specific points and be as accurate as possible. (No calculators)

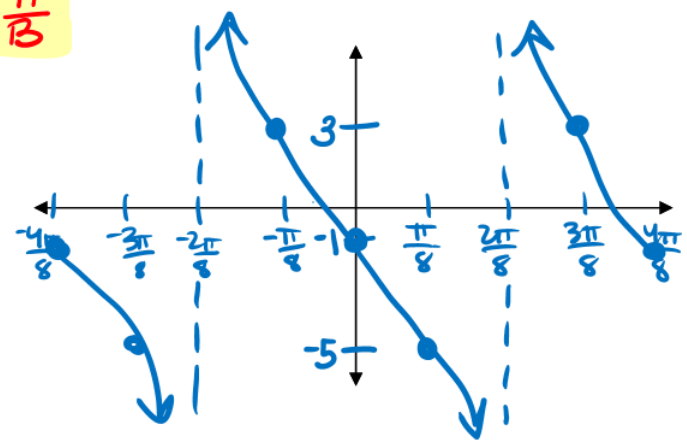
14) $y = -4\tan 2x - 1$

flip

Per: $\frac{\pi}{B}$

Per: $\frac{\pi}{2}$

Crit pts every $\frac{\pi}{8}$



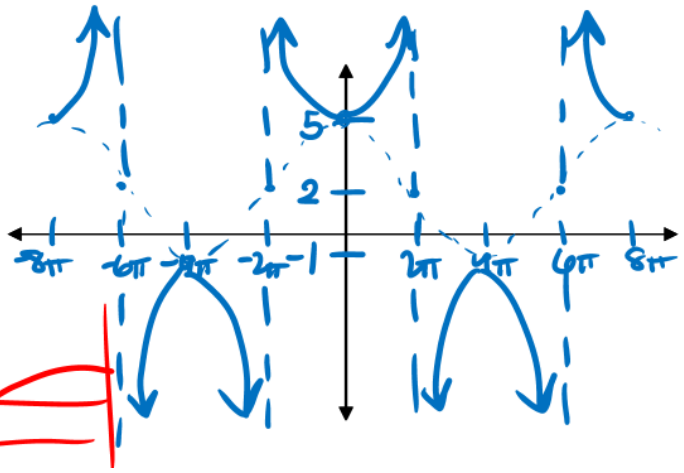
15) $y = 3\sec\frac{x}{4} + 2$

$\frac{1}{\cos}$

$B = \frac{1}{4}$

Per: $\frac{2\pi}{\frac{1}{4}} = 8\pi$

Crit pts 2π



Part VI: Find the exact values of each of the following. Remember to keep in mind the domains and ranges for the inverse functions. (No Calculator)

16) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\pi}{4}$

17) $\tan^{-1}(-1)$

$-\pi/4$

18) $\sec\left[\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

$\sec \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

19) $\sin\left[\tan^{-1}(1)\right]$

$\sin(\pi/4)$

1

Part VII: Use a calculator to find an approximate value (round to two decimal places and answer in radians).

20) $\sin^{-1}.8$

.93

21) $\cos^{-1}\frac{2}{5}$

1.16

22) $\cot^{-1}23$

$\tan^{-1}\left(\frac{1}{23}\right)$

.04

Find the domain and range of each:

1. $y = 4\sin(3x) - 2$

R $[-6, 2]$ D: $\mathbb{R}$

2. $y = -2\cos^{-1} 3(x - 0.5)$

D: $[1/6, 5/6]$

R: $[-\frac{\pi}{2}, 0]$

3. $y = \csc(.5x + \pi)\,4$

D: $\mathbb{R}$ $x \neq -2\pi + \frac{\pi}{2}k$

R: $(-\infty, -1] \cup [1, \infty)$

4. $y = 3\sec(2x) + 4$

D: $\mathbb{R}$ $x \neq \frac{\pi}{4} + \frac{\pi}{2}k$

R $(-\infty, -3] \cup [3, \infty)$

5. $y = -\tan(\pi x) - 1$

D: $\mathbb{R}$ $x = \frac{1}{2} + 1k$

R: $\mathbb{R}$

6. $y = 2\cot(2x) - 2$

D: $\mathbb{R}$ $x \neq \frac{\pi}{2}k$

R: $\mathbb{R}$

7. $f(x) = \frac{1}{4}\tan^{-1} \pi x - 1$

D: $\mathbb{R}$

R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

7b. Extension: $\lim_{x \to -\infty} f\ x = -\frac{\pi}{8}$