4.2 Day 1 — Implicit Differentiation (special case for the Chain Rule)

Thursday, October 5, 2017 10:51 AM

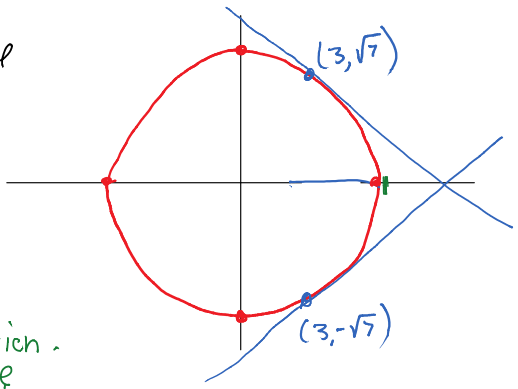
Notation Talk

$\frac{dy}{dx}$ the derivative of y w/ respect to x

$\frac{dv}{dt}$ the derivative of v w/ respect to t

$\frac{dx}{dx}$ the derivative of x w/ respect to x

Graph $x^2 + y^2 = 16$



$x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \pm\sqrt{16-x^2}$

$y = \sqrt{16-x^2}$ $\quad y = -\sqrt{16-x^2}$

$y = (16-x^2)^{1/2}$ $\quad y = -(16-x^2)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(16-x^2)^{-1/2}(-2x)$ $\quad \frac{dy}{dx} = -\frac{1}{2}(16-x^2)^{-1/2}(-2x)$

How do you find the rate of change of y w/ respect to x for the circle?

① Differentiate w/ respect x on both sides of the equation.
* Use the chain rule for all terms w/ y.

$$\frac{d}{dx}x^2 + \frac{d}{dx}(y)^2 = \frac{d}{dx}16$$

$$2x \cdot \left(1\frac{dx}{dx}\right) + 2(y)\cdot 1\frac{dy}{dx} = 0$$

$$2x + 2y\frac{dy}{dx} = 0$$

② Collect all the $\frac{dy}{dx}$ terms on one side and all the non-$\frac{dy}{dx}$ terms on the other side.

$$2y\frac{dy}{dx} = -2x$$

③ Factor out, when needed, the $\frac{dy}{dx}$ and isolate it.

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y} \qquad \frac{dy}{dx} = -\frac{x}{y}$$

Extend: Write the equation of the tangent line(s) @ $x = 3$ for $x^2 + y^2 = 16$.

P.o.t

$3^2 + y^2 = 16$

$9 + y^2 = 16$

$y^2 = 7$

$y = \pm\sqrt{7}$

$m = \frac{dy}{dx} = \frac{-x}{y}$

$(3, \sqrt{7})$ $\quad$ $(3, -\sqrt{7})$

$\frac{dy}{dx} = \frac{-3}{\sqrt{7}}$ $\quad$ $\frac{dy}{dx} = \frac{-3}{-\sqrt{7}} = \frac{3}{\sqrt{7}}$

$y - \sqrt{7} = \frac{-3}{\sqrt{7}}(x-3)$ $\quad$ $y + \sqrt{7} = \frac{3}{\sqrt{7}}(x-3)$

Ex 1. $x = xy$

$v \quad u' + u \quad v'$

$1 = y(1) + x \cdot \frac{dy}{dx}$

$1 - y = x\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1-y}{x}$

Ex 2: $y^2 = x^2 - y$

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$2y\frac{dy}{dx} = 2x\frac{dx}{dx} - \frac{dy}{dx}$

D. of out eval. @ the inside

$2y\frac{dy}{dx} + \frac{dy}{dx} = 2x$

$\frac{dy}{dx}(2y+1) = 2x$

$$\frac{dy}{dx} = \frac{2x}{2y+1}$$

Ex 3. Find $\frac{dy}{dx}$ of $x^2 - xy + y^2 = 7$

$$2x\frac{dx}{dx} - \left(\underset{v}{y}\cdot\frac{dx}{dx} + \underset{u}{x}\cdot\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x + y$$

$$\frac{\frac{dy}{dx}(-x+2y)}{-x+2y} = \frac{-2x+y}{-x+2y}$$

$$\frac{dy}{dx} = \frac{-2x+y}{-x+2y}$$