AP MC problems for Chapter 4.1-4.4 Name Key

D 1. $\frac{d}{dx}\cos^2(x^3) = \frac{d}{dx}(\cos(x^3))^2$

A) $6x^2\sin(x^3)\cos(x^3)$

B) $6x^2\cos(x^3)$

C) $\sin^2(x^3)$

D) $-6x^2\sin(x^3)\cos(x^3)$

E) $-2\sin(x^3)\cos(x^3)$

$= 2(\cos(x^3)) \cdot (-\sin(x^3)) \cdot 3x^2$

$= -6x^2\cos(x^3)(\sin(x^3))$

E 2. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

C) $y = 2\left(x - \frac{\pi}{4}\right)$

D) $y = -\left(x - \frac{\pi}{4}\right)$

E) $y = -2\left(x - \frac{\pi}{4}\right)$

$y = \cos\left(2\left(\frac{\pi}{4}\right)\right) = \cos\frac{\pi}{2} = 0 \quad \left(\frac{\pi}{4}, 0\right)$

$y' = -2\sin(2x) = -2\sin\frac{\pi}{2} = -2$

$y - 0 = -2\left(x - \frac{\pi}{4}\right)$

A 3. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point (4,3)?

A) $-\frac{25}{27}$ B) $-\frac{7}{27}$ C) $\frac{7}{27}$ D) $\frac{3}{4}$ E) $\frac{25}{27}$

$2x + 2yy' = 0 \quad y' = -\frac{x}{y} \quad y'' = \frac{y(-1) - (-x)y'}{y^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2} = \frac{-3 + 4\left(\frac{-4}{3}\right)}{9} \quad \frac{-9-16}{27} = \frac{-25}{27}$

B 4. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point (3,2)?

A) 0 B) $\frac{4}{9}$ C) $\frac{7}{9}$ D) $\frac{6}{7}$ E) $\frac{5}{3}$

$6yy' - 4x = -2xy' + -2y$

$6(2)y' - 4(3) = -2(3)y' - 2(2)$

$12y' - 12 = -6y' - 4$

$18y' = 8$

$y' = 8/18 = \frac{4}{9}$

E 5. If $f(x)=\frac{e^{2x}}{2x}$, then $f'(x)=$ $\frac{2x\cdot 2e^{2x}-e^{2x}\cdot 2}{4x^2}=\frac{2e^{2x}(2x-1)}{4x^2}=\frac{e^{2x}(2x-1)}{2x^2}$

A) 1

B) $\frac{e^{2x}(1-2x)}{2x^2}$

C) e^{2x}

D) $\frac{e^{2x}(2x+1)}{x^2}$

E) $\frac{e^{2x}(2x-1)}{2x^2}$

A 6. If $x^2+xy=10$, then when $x=2$, $\frac{dy}{dx}=$

$4+2y=10$ $2y=6$ $y=3$ $2x+y+xy'=0$ $2(2)+3+2y'=0$ $2y'=-7$ $y'=-7/2$

A) $-\frac{7}{2}$ B) -2 C) $\frac{2}{7}$ D) $\frac{3}{2}$ E) $\frac{7}{2}$

E 7. If $f(x)=\sin(e^{-x})$, then $f'(x)=$ $\cos(e^{-x})\cdot(-e^{-x})$

A) $-\cos(e^{-x})$

B) $\cos(e^{-x})+e^{-x}$

C) $\cos(e^{-x})-e^{-x}$

D) $e^{-x}\cos(e^{-x})$

E) $-e^{-x}\cos(e^{-x})$

A 8. If $f(x)=\ln(x+4+e^{-3x})$, then $f'(0)$ is

$f'(x)=\frac{1}{x+4+e^{-3x}}\cdot 1-3e^{-3x}=\frac{1-3e^{0}}{0+4+e^{0}}=\frac{-2}{5}$

A) $-\frac{2}{5}$ B) $\frac{1}{5}$ C) $\frac{1}{4}$ D) $\frac{2}{5}$ E) nonexistent

B 9. Let f be the function defined by $f(x)=x^3+x$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is the value of $g'(2)$?

A) $\frac{1}{13}$ B) $\frac{1}{4}$ C) $\frac{7}{4}$ D) 4 E) 13

$f'(1)=3+1=4$ $\quad g'(2)=\frac{1}{4}$

D 10. If $f(x)=e^{(2/x)}$, then $f'(x)=$

A) $2e^{(2/x)}\ln x$ B) $e^{(2/x)}$ C) $e^{(-2/x^2)}$ D) $-\frac{2}{x^2}e^{(2/x)}$ E) $-2x^2e^{(2/x)}$

$f'(x)=e^{\frac{2}{x}}\cdot\frac{-2}{x^2}$

A 11. If $f(x)=x^2+2x$, then $\frac{d}{dx}\left(f(\ln x)\right)=$

A) $\frac{2\ln x+2}{x}$ B) $2x\ln x+2x$ C) $2\ln x+2$ D) $2\ln x+\frac{2}{x}$ E) $\frac{2x+2}{x}$

$f(\ln x)=(\ln x)^2+2\ln x$ $\quad \frac{d}{dx}\left((\ln x)^2+2\ln x\right)=\frac{2\ln x}{x}+\frac{2}{x}$

D 12. If $\sin(xy)=x$, then $\frac{dy}{dx}=$

A) $\frac{1}{\cos(xy)}$

B) $\frac{1}{x\cos(xy)}$

C) $\frac{1-\cos(xy)}{\cos(xy)}$

D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

E) $\frac{y\left(1-\cos(xy)\right)}{x}$

$\cos(xy)(y+xy')=1$

$y+xy'=\frac{1}{\cos(xy)}$

$xy'=\frac{1}{\cos(xy)}-y$

$y'=\frac{1}{x\cos(xy)}-\frac{y}{x}$

$\Rightarrow \frac{1-y\cos(xy)}{x\cos(xy)}$