6.2 Day 1

Monday, April 22, 2019 7:41 AM

Dot Product $u = \langle u_1, u_2 \rangle$ $v = \langle v_1, v_2 \rangle$

$$u \cdot v = u_1 v_1 + u_2 v_2$$

example $u = \langle -2, 3 \rangle$ $v = \langle 4, 8 \rangle$

$u \cdot v =$

$-2(4) + (3)(8) = -8 + 24 = 16$

you try...

$u = 10i + 9j$ $v = -4i + 2j$

$u \cdot v = 10(-4) + 9(2) = -22$

Properties of the Dot Product:

let $u, v,$ & $w$ be vectors & $c$ be a scalar

1. $u \cdot v = v \cdot u$

2. $u \cdot u = |u|^2$

$|u| = \sqrt{u \cdot u}$

* use the dot product to find the length $\vec{u}$ $\langle 4, -3 \rangle$

$u \cdot u = 4(4) + (-3)(-3)$

$= 25$

$\sqrt{u \cdot u} = 5$ = length

3. $0 \cdot u = 0$

4. $u(v + w) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

5. $(c\vec{u})\vec{v} = \vec{u}(c\vec{v}) = c(\vec{u}\cdot\vec{v})$

☆ If the dot product = 0, then the vectors are orthogonal (perpendicular)

☆ 2 vectors are parallel if the have the same direction or are exactly opposite directions. vectors are parallel if one is a scalar multiple of the other.

Ex: If $\vec{u} = \langle 4,-3\rangle$ and $\vec{v} = \langle 7,k\rangle$ are orthogonal then find k.

$4(7) + -3k = 0$ $\qquad k = 28/3$

$28 \cancel{+} -3k = 0$

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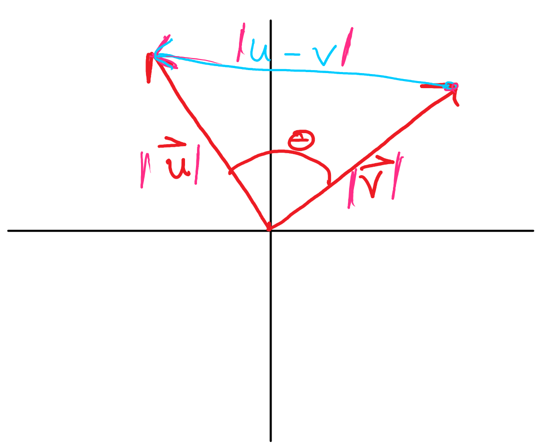
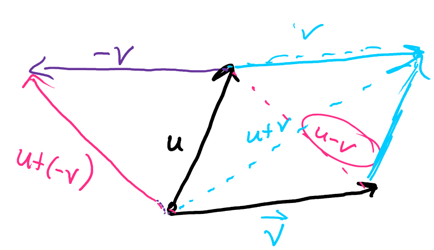
Angles between 2 vectors




$|u-v|^2 = |u|^2 + |v|^2 - 2|u||v|\cos\theta$

$(u-v)(u-v)$

$u\cdot u - 2u\cdot v + v\cdot v = |u|^2 + |v|^2 - 2|u||v|\cos\theta$

$\cancel{|u|^2} - 2u\cdot v + \cancel{|v|^2} = \cancel{|u|^2} + \cancel{|v|^2} - 2|u||v|\cos\theta$

$$-2u\cdot v = -2|u||v|\cos\theta$$

$$\cos\theta = \frac{u\cdot v}{|u||v|}$$

$$\theta = \cos^{-1}\left(\frac{u\cdot v}{|u|\,|v|}\right)$$

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Example! If $u=\langle 10, 9\rangle$ $|u|=\sqrt{181}$ $v=\langle -4, 2\rangle$ $\sqrt{20}$ find the angle between $u$ & $v$.

$$\cos\theta = \frac{10(-4)+9(2)}{\sqrt{181}\cdot\sqrt{20}} \qquad \theta = \cos^{-1}\left(\frac{-22}{\sqrt{181\cdot 20}}\right)$$

$$= 111^\circ$$

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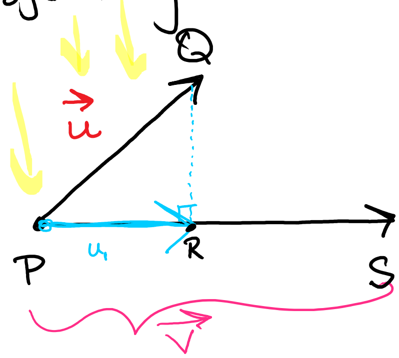
Projecting one vector onto another




$\mathrm{Proj}_{v}u$

The vector projection of $\vec{u}=\overrightarrow{PQ}$ onto a non-zero vector $\vec{v}=\overrightarrow{PS}$ is the vector $\overrightarrow{PR}$ determined by dropping a $\perp$ from Q to the PS.

$$\vec{u} = \overrightarrow{PR} + \overrightarrow{RQ} \qquad RQ = u - \overrightarrow{PR} \text{ or } u - u_1$$

$$\mathrm{Proj}_v u = \underbrace{\left(\frac{u\cdot v}{|v|^2}\right)}_{\text{scalar}} v$$

* Decomposing a vector into Perpenicular components

$$(u\cdot v)\left(\frac{v}{|v|}\right)$$