Derivatives of Trig Functions

$$\boxed{\frac{d}{dx}\sin x = \cos x} \qquad \boxed{\frac{d}{dx}\cos x = -\sin x}$$

Given $y = \overset{u}{x^3} \cdot \overset{v}{\cos x}$ find y'

$y' = \cos x \cdot 3x^2 + x^3(-\sin x)$

$y' = 3x^2\cos x - x^3\sin x$

you try ... $\dfrac{d}{dx}\dfrac{\sin x}{2-\cos x}$ $\dfrac{u}{v}$ $= \dfrac{(2-\cos x)(\cos x) - \sin x(\sin x)}{(2-\cos x)^2}$

$$= \frac{2\cos x - (\cos^2 x - \sin^2 x)}{(2-\cos x)^2} = \frac{2\cos x - 1\,(\overbrace{\cos^2 x + \sin^2 x})}{(2-\cos x)^2}$$

$$= \frac{2\cos x - 1}{(2-\cos x)^2}$$

$\sin x \cdot (-\sin x)$
$-(-\sin x \cdot \sin x)$

$f(x) = \tan x$ find $f'(x)$

$f(x) = \dfrac{\sin x}{\cos x}$ $\dfrac{u}{v}$ $\qquad f'(x) = \dfrac{\overset{v}{\cos x}\,\overset{u'}{(\cos x)} - \overset{u}{\sin x}\,\overset{v'}{(-\sin x)}}{\cos^2 x}$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\boxed{\frac{d}{dx}\tan x = \sec^2 x} \qquad \boxed{\frac{d}{dx}\cot x = -\csc^2 x}$$

Given $y = \sec x$ find $\dfrac{dy}{dx}$

$y = \dfrac{1}{\cos x}$ $\qquad \dfrac{dy}{dx} = \dfrac{\cos x\,(0) - 1(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x} = \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x}$

$$\boxed{\frac{d}{dx}\sec x = \sec x\tan x} \qquad \boxed{\frac{d}{dx}\csc x = -\csc x\cot x}$$

$\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$ $\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$ $\frac{d}{dx}\csc x = -\csc x \cot x$

Find the first four derivatives of $y = \sin x$.

$y' = \cos x$

$y'' = -\sin x$

$y''' = -\cos x$

$y^{(4)} = \sin x$

$y^{(102)} = y'' = -\sin x$

$102 \div 4 = 25$ R 2

Write the equation of the tangent line to $f(x) = \cos x$ at $x = \frac{2\pi}{3}$.

$y - y_1 = m(x - x_1)$

☆ $f\left(\frac{2\pi}{3}\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$ p.o.t $\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$

$\boxed{y + \frac{1}{2} = -\frac{\sqrt{3}}{2}\left(x - \frac{2\pi}{3}\right)}$

$m = f'(x) = -\sin x$

$m = f'\left(\frac{2\pi}{3}\right) = -\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2} = m$