3.4 and 3.5 (Tues. 9/17)

Monday, September 16, 2019 10:53 AM

Calculus AB **Name**________________________

3.4-3.5 Notes Day 1 and 2

3.4: Position, Velocity, and Acceleration

Suppose an object is moving along a linear path. We will use feet for distance and seconds for time throughout the examples that follow.

I. Let $s(t)$ be the **position** of the object at time *t*. Think of it as measuring your position compared to "home" at s=0.

Ex: s(2)=50 means that after 2 seconds you are 50feet ahead of home.

- $s(b)-s(a)$ is the **displacement** during the time interval [a,b].

Ex: s(2) – s(0) = -30 means that you traveled 30ft backwards during the 0-2 second time period. (So s(0) was 80ft).

- $\dfrac{s(b)-s(a)}{b-a}$ is **avg velocity** (avg rate of Δ in position) during the interval [a,b]

II. $s'(t)=v(t)$ is the **(instantaneous) velocity** of the object at time t.

Ex: s'(5) = -20 means that at t = 5 seconds, you are going <u>backwards</u> at 20 ft/sec

- $|v(t)|$ is the **speed** at time *t*. It's the same as velocity, except it ignores direction.

Ex: $|s'(5)|=|v(5)|=20$ *means you were moving 20 ft/sec at t = 5 seconds.*

III. $s''(t)=v'(t)=a(t)$ is the **acceleration** at time *t*. It measures how quickly your velocity changes.

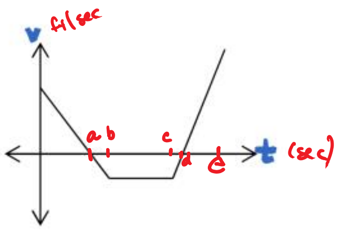
Ex: a(5) = -3 means that at t = 5 seconds your velocity is decreasing by 3ft/sec^2. Since your velocity is -20 ft/sec at that time, decreasing the velocity makes it more negative, and actually makes your speed increase. You're going backwards, and you are speeding up in that direction.

<u>Speeding up</u> - velocity & acceleration have the same signs

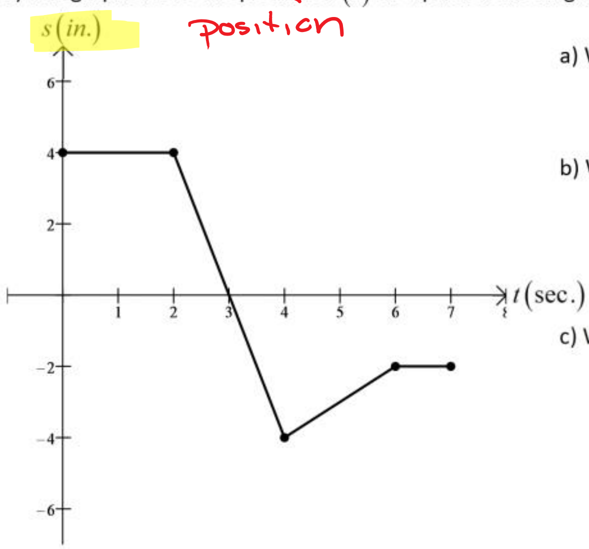
<u>Slowing Down</u> – velocity & acceleration have opposite signs

IV. $s'''(t)=v''(t)=a'(t)=j(t)$ is called Jerk which is the rate of change of acceleration.

Example: Describe the position of the smiley face using its velocity graph.



1) The graph shows the position $s(t)$ of a particle moving along a horizontal coordinate axis.



a) When is the particle moving to the left?

(2,4)

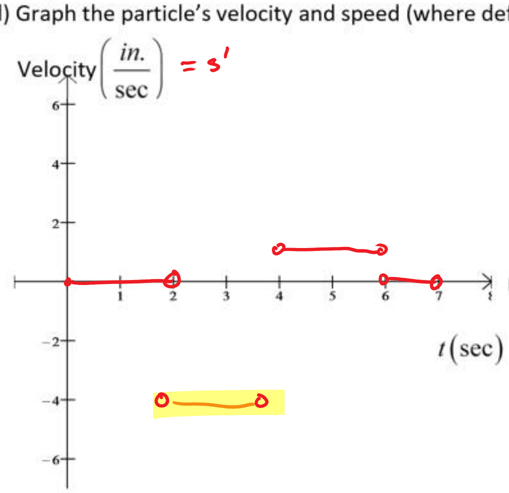
b) When is the particle moving to the right?

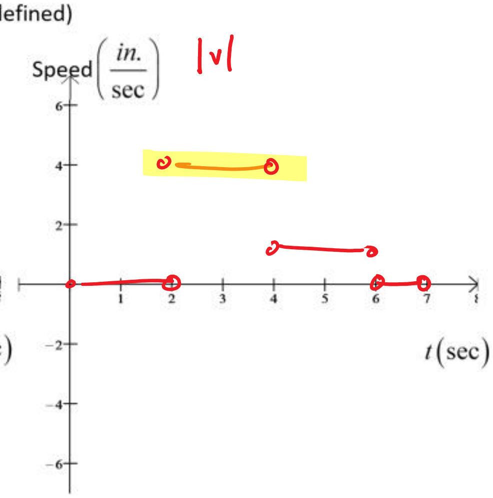
(4,6)

c) When is the particle standing still?

$(0,2) \cup (6,7)$

d) Graph the particle's velocity and speed (where defined)

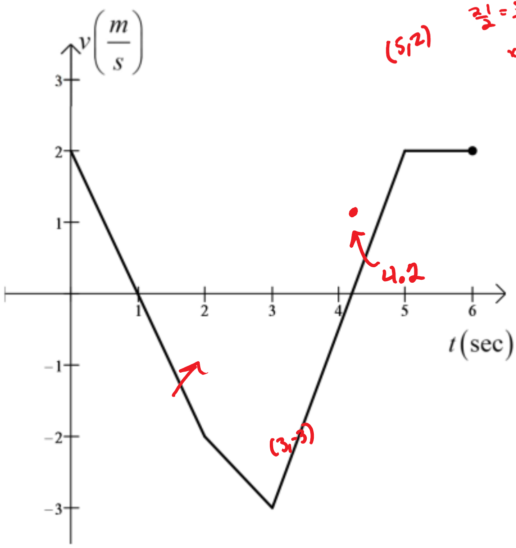




e) When is the particle moving the fastest?

2) The graph shows the velocity $v = f(t)$ of a particle moving along a horizontal coordinate axis.

$m = \frac{5}{2}$ $y - 2 = \frac{5}{2}(x-5)$
$-2 = \frac{5}{2}x - \frac{25}{2}$
$\frac{21}{2} = \frac{5}{2}x$
$x = 21/5$
$(5,2)$
$v(t) < 0$
$(1, 4.2)$



a) When does the particle reverse direction?

$(5,6)$

b) When is the particle moving at a constant speed?

$t = 3$ secs

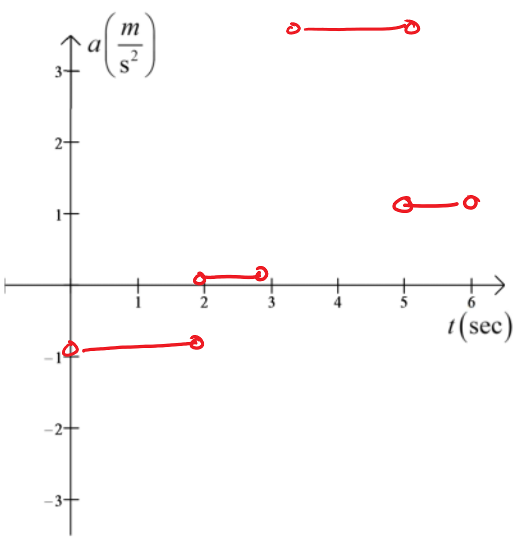
c) When is the particle moving at its greatest speed?

Extend: when does the particle change direction? Justify.

$t = 1$ & 4.2

B/c velocity changes signs @ $t = 1$ & 4.2

d) Graph the acceleration (where defined)



Extend: Is the particle speeding up or down @ $t = 2.5$ secs?

• speeding up b/c $v(2.5) < 0$ and v is decreasing

• speeding up b/c $v(2.5) < 0$ and $v'(2.5) = a(2.5) < 0$

3) A particle moves along a vertical coordinate axis so that its position at any time $t \geq 0$ is given by the

3) A particle moves along a vertical coordinate axis so that its position at any time $t \geq 0$ is given by the function $s(t) = \frac{1}{3}t^3 - 3t^2 + 8t - 4$, where *s* is measured in cm and *t* is measured in seconds.

a) Find the displacement during the first 6 seconds.

$S(6) - S(0) =$
$8 - (-4) = 12$ ft

b) Find the average velocity during the first 6 seconds.

$\frac{S(6) - S(0)}{6 - 0} = \frac{12}{6}$ 2cm / ft.

c) Find expressions for the velocity and acceleration at time t.

$v(t) = t^2 - 6t + 8$

$a(t) = \underline{2t - 6}$

d) For what values of *t* is the particle moving downward? $v(t) < 0$

$(t-4)(t-2) < 0$

	- -		- +		+ +	v(t)
0	P	2	N	4	P	

(2,4) b/c $v(t) < 0$ over the interval.

Extend: Is the partical speeding up or down @ $t = 3.5$ secs? Justify

$V(3.5) < 0$ $a(3.5) > 0$

the particle is Slowing down ~~speeding~~ @ $t = 3.5$
b/c $v(t)$ & $a(t)$ have opposite signs @ $t = 3.5$.

3.5 Derivatives of Trigonometric Functions

Trig Derivatives – Let's look at the graphs for sinx and cosx!

$\frac{d}{dx}\sin x =$ cosx

$\frac{d}{dx}\cos x =$ −sinx

Now let's use these two derivatives to find the other ones!

$\frac{d}{dx}\tan x = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \sec^2 x$

$= \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

$\frac{d}{dx}\sec x = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \tan x \sec x$

$= \frac{\cos x(0) - 1(-\sin x)}{\cos^2 x}$

$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$= \tan x \sec x$

Ladies: $\frac{d}{dx}\cot x = -\csc^2 x$

Gentlemen: $\frac{d}{dx}\csc x = -\cot x \csc x$

1. Write the equation of the tangent line to $y = 2x + \sin x$ @ $x = \pi$.

$\frac{dy}{dx} = \frac{d}{dx}2x + \frac{d}{dx}\sin x$

$= 2 + \cos x$

p.o.t $y = 2\pi + \sin\pi$

$= 2\pi$

p.o.t $(\pi, 2\pi)$

$\boxed{y - 2\pi = 1(x - \pi)}$

★ $m = y' = 2 + \cos x$

$y' = 2 + \cos\pi$

$= 2 - 1 = 1$

$\frac{d}{dx}(u \cdot v) = u \cdot v' + v \cdot u'$

2. Find $\frac{d}{dx}(x \cdot \tan x)$ $= x \cdot \sec^2 x + \tan x(1)$

$= x\sec^2 x + \tan x$

3. Find the jerk at time t if s(t) = 2t – 2cost.

$s'(t) = 2 + 2\sin t$

$s''(t) = 2\cos t$

$s'''(t) = -2\sin t$

Closer:

Find the rate of change of the volume of the sphere with respect to the length of its radius. Then, find the rate of change when the radius is 3 inches.

Start with the formula for volume of a sphere:

Find the first derivative of the volume w.r.t. its radius:

Evaluate at r = 3:

Supply the correct units: