Series Exploration KEY

**Calculus BC** Name Me

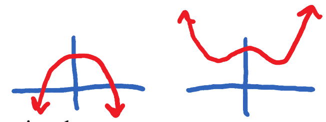
**Series Introductory Exploration**

Recall from your previous courses that the sum of an infinite geometric series is defined as $S_\infty = \dfrac{a}{1-r}$ where $a$ is the first term and $r$ is the common ratio between terms.

1) Find the sum of the infinite series $9 + 3 + 1 + \frac{1}{3} + \ldots$ $= \dfrac{9}{1-\frac{1}{3}} = \dfrac{27}{2}$

2) What is the restriction on $r$ that guarantees a finite sum? $|r| < 1$

3) Go to www.desmos.com, then graph $y = 1 - x^2 + x^4 - x^6$. Observe its shape.



4) By continuing the pattern of terms, add one more term and graph again. Observe its shape.

5) Add the next term and graph again. What are your observations of the effects of adding each term?

- End behavior flips w/each new term
- Interval near y-int. is relatively consistent

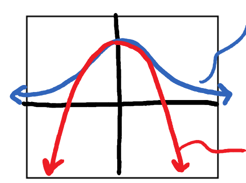
6) Think of the polynomial as a geometric series. $\rightarrow \sum_{n=1}^{\infty} a_n = 1 - x^2 + x^4 - x^6 + \ldots$

a. What is the common ratio? $r = -x^2$

b. If it were infinite, what would its "sum" be? (Hint: This will a rational function in terms of x.)

$S = \dfrac{a}{1-r} = \dfrac{1}{1+x^2}$

7) Graph both the "sum" ($\rightarrow \infty$ # terms) and the polynomial ($\rightarrow$ finite # terms).
Sketch the polynomial and its sum to the right.



$S = \dfrac{1}{1+x^2}$ Infinite Geometric Sum

Finite Polynomial (Partial Sum)

8) By adding more terms to the polynomial, see how far you can expand the domain over which the polynomial matches the sum expression. Does there seem to be a limitation to the width of the domain? If so, what might be causing that limitation? (Hint: Use question 2.)

Interval of convergence limited to $-1 < x < 1$

Why? Need $|r| = |-x^2| < 1$

8) Click on the colored symbol in front of each of your functions from the previous questions to turn off the graphs.

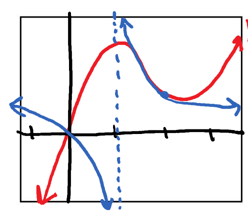
9a) Think of $\frac{x}{x-1}$ as the sum of an infinite geometric series. Try to determine a first term, a common ratio, and write the first five terms of the series. Hint: Rewrite the given sum in the form: $\frac{x}{1-(\text{expression})}$

$$\frac{a}{1-r} = \frac{x}{x-1} = \frac{x}{1-(2-x)} = x + x(2-x) + x(2-x)^2 + x(2-x)^3 + x(2-x)^4 + \dots$$

• let $a = x$
• let $1-r = x-1$
$r = 1-(x-1)$
$= 2-x$

• $y_1 = \frac{x}{1-(2-x)} = \frac{x}{x-1} = \infty$ series

• $y_2 = x + x(2-x) + x(2-x)^2 + x(2-x)^3 + x(2-x)^4$
$=$ finite polynomial (partial sum)



$Y_2 = \sum_{n=0}^{4} x(2-x)^n$

$Y_1 = \sum_{n=0}^{\infty} x(2-x)^n$

Graph the given sum, and the series you just wrote above.
Sketch them ~~below~~. over here →

Over what approximate domain do the sum and the series match?

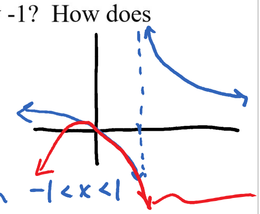
$1 < x < 3$

Interval of convergence is centered @ $x=2$.

9b) How is the result different if you multiply the numerator and denominator of $\frac{x}{x-1}$ by -1? How does this effect the domain (interval of convergence) where the sum and series match?

$$\frac{x}{x-1} = \frac{-x}{1-x} = -x - x^2 - x^3 - x^4 - x^5 - \dots$$

$= \frac{a}{1-r}$



$Y_1 = \frac{-x}{1-x}$

$Y_2 = -x - x^2 - \dots - x^4$

Polynomial converges to $\infty$ series on $-1 < x < 1$

Int. of convergence is centered @ $x = 0$.

11) Add more terms to the polynomial to try to expand the interval of convergence (where the polynomial series converges to the sum expression). Do they converge over their entire domains? Why or why not?

No b/c $|r| = |2-x|$ must be $< 1$

$|2-x| < 1$

$-1 < 2-x < 1$

$-3 < -x < -1$

$3 > x > 1$

12) Click on the colored symbols in front of each of the graphs you just graphed to turn them off.

13) Explore the graph of the following non-geometric series. Try adding terms one at a time. (Make sure to add LOTS of terms. Try to go until the least the 13th power.)

$$x-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots$$

Whoa...

a. If we added on infinite terms to this series, the sum converges to a function you know. What function does that sum appear to be? Test it by graphing that function on Desmos.

$= \sin x$ ?!?!?

b. Comment on the interval of convergence. (The interval where the function and sum are converging to be the same thing.)

Converges $(-\infty, \infty)$

14) Take the derivative of the series in part 13 term by term and examine its graph. What happened? What function does this polynomial's sum appear to be approaching?

$$\frac{d}{dx}\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots\right)$$

$$=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\dots = \cos x$$ ?!?!