3.1 Day 1

Wednesday, August 28, 2019 8:19 AM

Derivatives

Def: The derivative of a function $f(x)$ with respect to x is the function $f'(x)$ whose value at x is:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

★ The function $f'(x)$ outputs slopes of $f(x)$ at any point x on the curve.

★ If the derivative exists, then we say the function is differentiable.

Ex: Differentiate $f(x) = x^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2 + 3x(0) + (0)^2$$

$$f'(x) = 3x^2$$

a. Find the slope of f @ $x = 2$

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$$f'(2)=3(2)^2=12$$

b. Find the equation of the tangent line to $f(x)$ @ $x=3$.

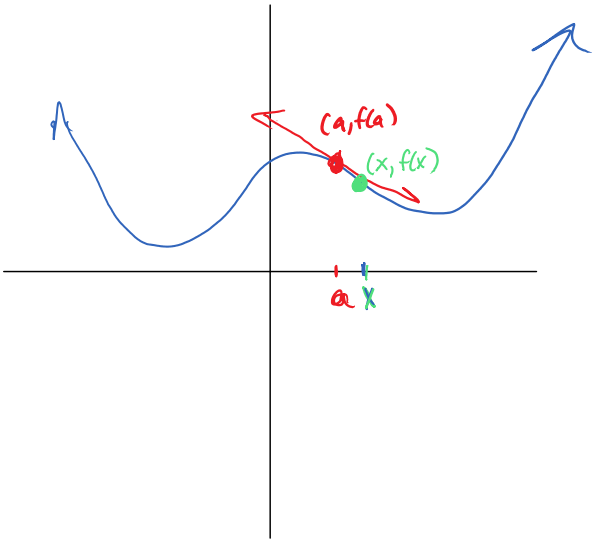
$f(3)=3^3=27$ $(3,27)$ p.o.t $m=f'(3)=3(3)^2=27$

$$y-27=27(x-3)$$

Another def. of a derivative

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

is the derivative of f @ $x=a$



Ex: $f(x)=\frac{1}{x}$ find $f'(a)$

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$=\lim_{x\to a}\frac{\frac{1}{x}-\frac{1}{a}}{x-a}$$

$$=\lim_{x\to a}\frac{\frac{a-x}{ax}}{x-a}$$

$$\lim_{x\to a}\frac{a-x}{ax}\cdot\frac{1}{x-a}$$

$$\lim_{x\to a}\frac{-(x-a)}{ax}\cdot\frac{1}{(x-a)}$$

$$=-\frac{1}{a\cdot a}=-\frac{1}{a^2}$$

$$f'(a)=\frac{-1}{a^2}$$

a. Write the eqtn. of the normal to $f(x)$ at $x=2$.

$f(2)=\frac{1}{2}$ } ap. of pt. $f'(2)=\frac{-1}{2^2}=-\frac{1}{4}$ } slope of tangent line

$$y-\frac{1}{2}=4(x-2)$$

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you try.... Differentiate $f(x)=x^4-3$

$$f'(a)=\lim_{x\to a}\frac{(x^4-3)-(a^4-3)}{x-a}$$

$$=\lim_{x\to a}\frac{x^4-a^4}{x-a}$$

$$=\lim_{x\to a}\frac{(x^2-a^2)(x^2+a^2)}{(x-a)}$$

$$=\lim_{x\to a}\frac{(x-a)(x+a)(x^2+a^2)}{(x-a)}$$

$$=(a+a)(a^2+a^2)$$

$$=2a(2a^2)=4a^3$$

$f'(x)=4x^3$

Notation's

$f'(x)\Rightarrow$ derivative of f w/ respect to x.

y' => brief, doesn't indicate the indep. variable

$\frac{dy}{dx}$ => derivative of y w/ respect to x

$\frac{df}{dx}$ => derivative of f w/ respect to x

$\frac{d}{dx}[f(x)]$ => derivative of f w/ respect to x

Ex $\frac{d}{dx}(x^3) \Rightarrow 3x^2$