**AP Calculus AB** **3.1 – 3.5 AP Review Questions** **Name ________________________**

1. (no calculator)

Let $f(x) = 3 + 2\sin x$

(a) Write an equation of the line tangent to the graph of f at $x = \frac{\pi}{6}$.

Point : $f\left(\frac{\pi}{6}\right) = 3 + 2\sin\frac{\pi}{6}$
$= 3 + 2\left(\frac{1}{2}\right)$
$= 4$
$\left(\frac{\pi}{6}, 4\right)$

Slope: $f'(x) = 2\cos x$
$f'\left(\frac{\pi}{6}\right) = 2\cos\frac{\pi}{6}$
$= 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

$\boxed{y - 4 = \sqrt{3}\left(x - \frac{\pi}{6}\right)}$

(b) Find the values of x in [o, 2π) for which the graph of f has a horizontal tangent.

$f'(x) = 2\cos x = 0$
$\cos x = 0$
$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

(c) Find $f''(x)$.

$\boxed{f''(x) = -2\sin x}$

2. (no calculator)

Let $f(x) = 2x^3 - x$ and $g(x) = \frac{f(x)}{x}$.

(a) What is the slope of the graph of f at x = -1? Show the work that leads to your answer.

$f'(x) = 6x^2 - 1$
$f'(-1) = 6(-1)^2 - 1 = 5$

(b) Write an equation of the line tangent to the graph of g at x = -1. (Quotient Rule)

Point $g(-1) = \frac{2(-1)^3 - (-1)}{-1}$
$g(-1) = 1$
$(-1, 1)$

Slope $g'(x) = \frac{x(6x^2 - 1) - (2x^3 - x)(1)}{x^2}$
$= \frac{6x^3 - x - 2x^3 + x}{x^2} = \frac{4x^3}{x^2} = 4x$
$g'(-1) = 4(-1) = -4$

$\boxed{y - 1 = -4(x + 1)}$

(c) What is the slope of the line normal to the graph of g at x = -1?

$\perp$ slope $= \boxed{\frac{1}{4}}$

**3.** (no calculator)

Evaluate each limit analytically.

(*Note: Finding the answer should* ***not*** *involve a lengthy algebraic process.*)

(a) $\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$

(b) $\lim_{h\to 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}$

(c) $\lim_{h\to 0} \frac{\sqrt{16+h} - 4}{h}$

(d) $\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$

(a) $f(x) = \sin x \quad f'(x) = \boxed{\cos x}$

(b) $f(x) = \sqrt[3]{x} = x^{1/3} \quad f'(x) = \boxed{\frac{1}{3}x^{-2/3}}$

(c) $f(x) = \sqrt{x} = x^{1/2}$ at $x = 16 \quad f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f'(16) = \frac{1}{2\sqrt{16}} = \boxed{\frac{1}{8}}$

(d) $f(x) = \frac{1}{x}$ at $x = 5 \quad f'(x) = -\frac{1}{x^2}$

$f'(5) = \frac{-1}{(5)^2} = \boxed{\frac{-1}{25}}$

**4.** Given:

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 2 | $-3$ | 1 | 5 | $-2$ |
| 5 | 4 | 7 | $-1$ | 2 |

(a) If $h(x) = \frac{f(x)}{g(x)}$, find $h'(2)$.

$$h'(x) = \frac{g(x)\cdot f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(2) = \frac{g(2)\cdot f'(2) - f(2)g'(2)}{[g(2)]^2} = \frac{5\cdot 1 - (-3)\cdot(-2)}{5^2} = \frac{5-6}{25} = \boxed{\frac{-1}{25}}$$

(b) If $j(x) = f(x)\bullet g(x)$, find j'(5).

$$j'(x) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

$$j'(5) = f(5)\cdot g'(5) + g(5)\cdot f'(5)$$

$$= 4\cdot 2 + (-1)\cdot(7)$$

$$= 8 - 7 = \boxed{1}$$

**5.** (no calculator)

Given: $f(x) = x^2$

(a) Find the slope of the normal line to the graph of $f$ at $x = -3$.

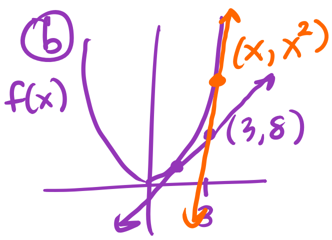
(b) Two lines passing through the point (3, 8) will be tangent to the graph of $f$. Find an equation for each of these lines.

a) $f'(x) = 2x$

$f'(-3) = 2(-3) = -6$

$\perp m = \boxed{\frac{1}{6}}$

b)




$f'(x) = 2x =$ slope of tangent lines

$2x = \dfrac{x^2 - 8}{x - 3}$ slope between (3, 8) and $(x, x^2)$ ← f(x)

$2x^2 - 6x = x^2 - 8$

$x^2 - 6x + 8 = 0$

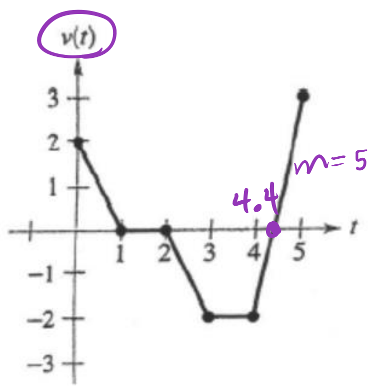
$(x-2)(x-4) = 0$

$x = 2, 4$

Point (2, 4) Slope $f'(2) = 4$ $\boxed{y - 4 = 4(x-2)}$

Point (4, 16) slope $f'(4) = 8$ $\boxed{y - 16 = 8(x-4)}$

**6.** The accompanying diagram shows the graph of the velocity in $\frac{\text{ft}}{\text{sec}}$ for a particle moving along the line $x = 4$.




(a) During which time interval is the particle:

(i) moving upward? $[0, 1) \cup (4.4, 5]$

(ii) moving downward? $(2, 4.4)$

(iii) at rest? $[1, 2] \cup [4.4]$

(b) State the acceleration of the particle at the specified times. Include units.

$a(t) = v'(t) =$ slope of velocity graph

(i) $t = 0.75$ $a(.75) = -2$ ft/sec$^2$

(ii) $t = 4.2$ $a(4.2) = 5$ ft/sec$^2$

**7.** (no calculator)

Given: $g(x) = f(x) \cdot \tan x + kx$, where $k$ is a real number.

$f$ is differentiable for all $x$; $f\left(\frac{\pi}{4}\right) = 4$; $f'\left(\frac{\pi}{4}\right) = -2$.

(a) For what values of $x$, if any, in the interval $0 < x < 2\pi$ will the derivative of $g$ fail to exist? Justify your answer.

(b) If $g'\left(\frac{\pi}{4}\right) = 6$, find the value of $k$.

(a) g(x) has infinite discontinuities at $x = \frac{\pi}{2}, \frac{3\pi}{2}$ in $(0, 2\pi)$ so $g'(x)$ will not exist at $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

(b) $g'(x) = f(x) \cdot \sec^2 x + f'(x)\tan x + k$

$g'(\frac{\pi}{4}) = f(\frac{\pi}{4}) \cdot \sec^2(\frac{\pi}{4}) + f'(\frac{\pi}{4})\tan\frac{\pi}{4} + k$

$= 4 \cdot (\sqrt{2})^2 + (-2)(1) + k = 6$

$8 - 2 + k = 6$ $\boxed{k = 0}$

**8.** The table provided below shows the position of a particle, $S$, at several times, $t$, as the particle moves along a straight line, where $t$ is measured in seconds and $S$ is measured in meters.

| $t$ | 2.0 | 2.7 | 3.2 | 3.8 |
|---|---|---|---|---|
| $S(t)$ | 5.2 | 7.8 | 10.6 | 12.2 |

Which of the following best estimates the velocity of the particle at $t = 3$?

(a) $9.2\frac{\text{m}}{\text{s}}$ (b) $7.8\frac{\text{m}}{\text{s}}$ (c) $5.6\frac{\text{m}}{\text{s}}$

avg vel $= \frac{10.6 - 7.8}{3.2 - 2.7} = \frac{2.8}{.5} = 5.6$