Calculus
Review 3.1-3.3
No Calculator

Name Key

(1-2) Use the definition of a derivative to find the following:

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \quad or \quad f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

1. if $f(x)=\sqrt{2x-1}$ *find* $f'(5)$

$$f'(5)=\lim_{x\to 5}\frac{\sqrt{2x-1}-3}{x-5}\cdot\frac{\left(\sqrt{2x-1}+3\right)}{\left(\sqrt{2x-1}+3\right)}$$

$$=\lim_{x\to 5}\frac{2x-1-9}{(x-5)\left(\sqrt{2x-1}+3\right)}$$

$$=\lim_{x\to 5}\frac{2x-10}{(x-5)\left(\sqrt{2x-1}+3\right)}$$

$$\lim_{x\to 5}\frac{2\cancel{(x-5)}}{\cancel{(x-5)}\left(\sqrt{2x-1}+3\right)}$$

$$=\frac{2}{\sqrt{9}+3}=\frac{2}{6}=\frac{1}{3}$$

2. if $f(x)=\dfrac{1}{2x-1}$ *find* $f'(x)$

$$f'(x)=\lim_{h\to 0}\frac{\frac{1}{2(x+h)-1}-\frac{1}{2x-1}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{2x+2h-1}-\frac{1}{2x-1}}{h}$$

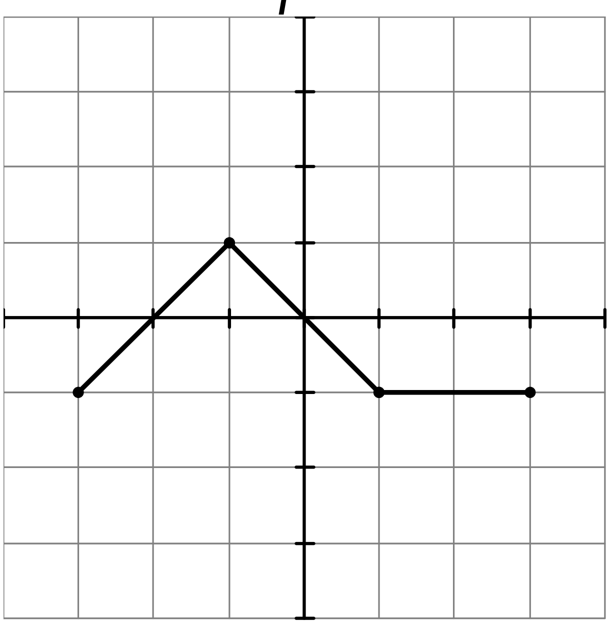
$$=\lim_{h\to 0}\frac{\frac{\cancel{2x}-\cancel{1}-\cancel{2x}-2h+\cancel{1}}{(2x-1)(2x+2h-1)}}{h}$$

$$=\lim_{h\to 0}\frac{-2\cancel{h}}{(2x-1)(2x+2h-1)}\cdot\frac{1}{\cancel{h}}=\frac{-2}{(2x-1)^2}$$

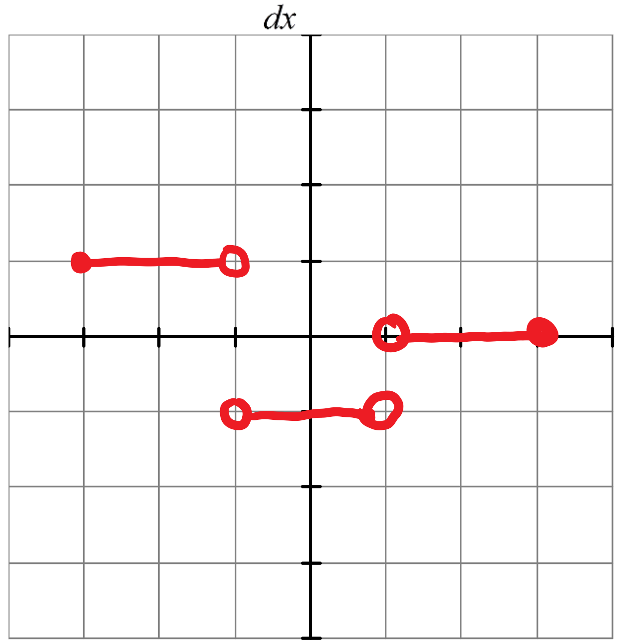
Given $f(x)$**, sketch** $\dfrac{df}{dx}$

3)

f



$\dfrac{df}{dx}$



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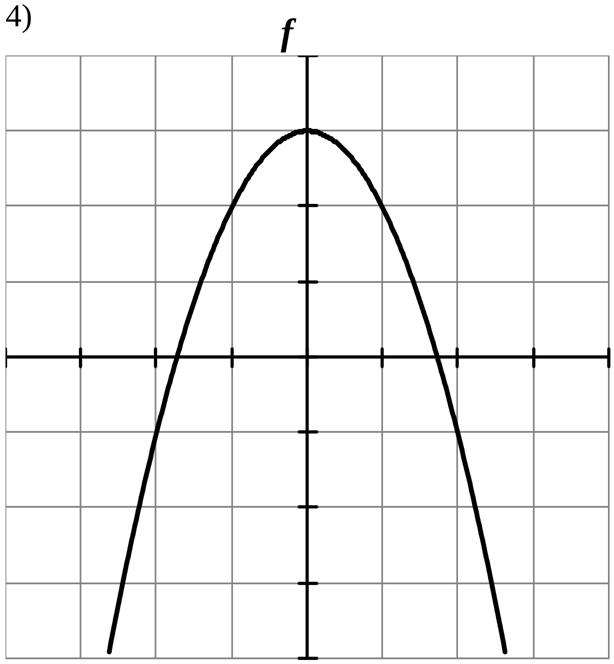
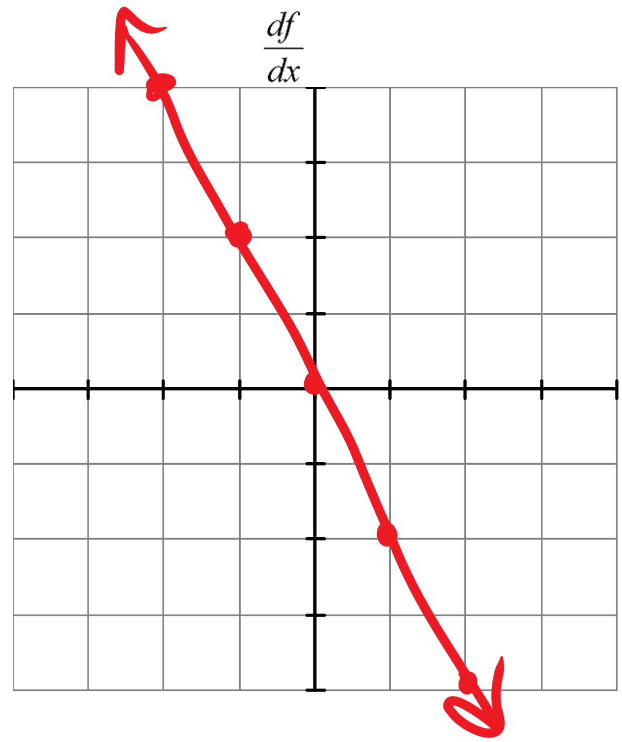
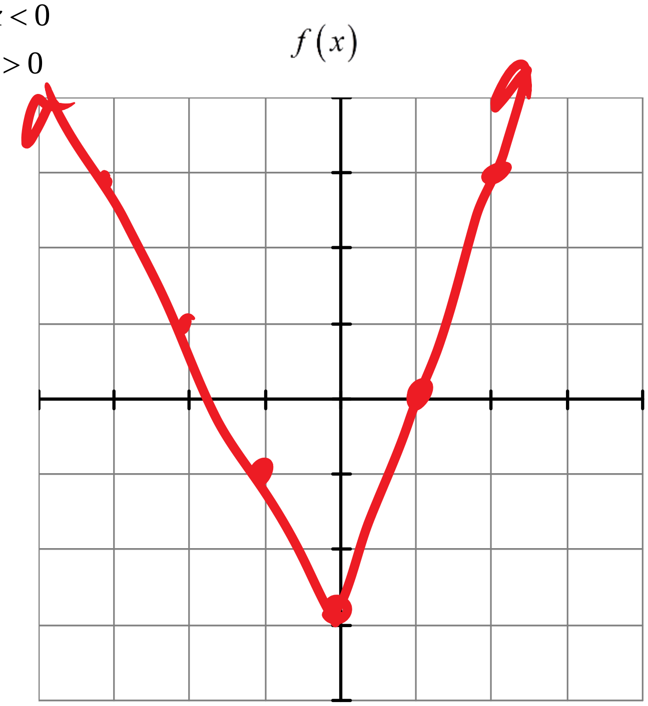




5. Sketch the graph of the **continuous** function $f(x)$ that has the following properties:

$f(2) = 3$ and $f'(x) = \begin{cases} -2 & \text{for } x < 0 \\ 3 & \text{for } x > 0 \end{cases}$



6. $f(x) = 8x^5 - 6x^3 + \frac{3}{4}x^2 + 5$ find $f'(x)$

$f'(x) = 40x^4 - 18x^2 + \frac{3}{2}x$

7. Find $\frac{dy}{dt}$ if $y = \frac{4t^2 - t}{3t+2}$

$\frac{dy}{dt} = \frac{(3t+2)(8t-1) - (4t^2-t)(3)}{(3t+2)^2}$

$= \frac{24t^2 + 16t - 3t - 2 - 12t^2 + 3t}{(3t+2)^2}$

$\frac{dy}{dt} = \frac{12t^2 + 16t - 2}{(3t+2)^2}$

8. Evaluate: $\frac{d}{dx}\left((3x-1)(5x^4 - 2x + 4)\right)$

$= (5x^4 - 2x + 4)(3) + (3x-1)(20x^3 - 2)$

$= 15x^4 - 6x + 12 + 60x^4 - 6x - 20x^3 + 2$

$= 75x^4 - 20x^3 - 12x + 14$

9. Write the equation of the tangent line to $g(x) = \frac{1-5x}{2x}$ at $x = 3$.

$g(3) = \frac{1-5(3)}{6} = \frac{-14}{6} = \frac{-7}{3}$

$g'(x) = \frac{2x(-5) - (1-5x)(2)}{(2x)^2}$

$g'(3) = \frac{6(-5) - (1-15)(2)}{6^2}$

$= \frac{-30 + 28}{36} = -\frac{1}{18}$

$y + 7/3 = \frac{-1}{18}(x-3)$

10. Where does $h(x) = \frac{1}{3}x^3 - \frac{x^2}{2} + 1$ have horizontal tangents?

$h'(x) = 0$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$ & $x = 1$

11) Let $h(x) = f(x) \cdot g(x)$ and $j(x) = \dfrac{f(x)}{g(x)}$. Fill in the missing entries in the table below using the information about f and g given and the definitions of h and j.

$h'(x) = g(x) \cdot f'(x) + f(x) \cdot g'(x)$

$j'(x) = \dfrac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h'(x)$	$j'(x)$
-2	1	-1	-3	4	7	$\frac{-1}{9}$
-1	0	-2	1	1	-2	-2
0	-1	2	-2	1	-5	-3/4

$h'(2) = g(-2) \cdot f'(-2) + f(-2) \cdot g'(-2)$
$= -3(-1) + 1 \cdot (4)$

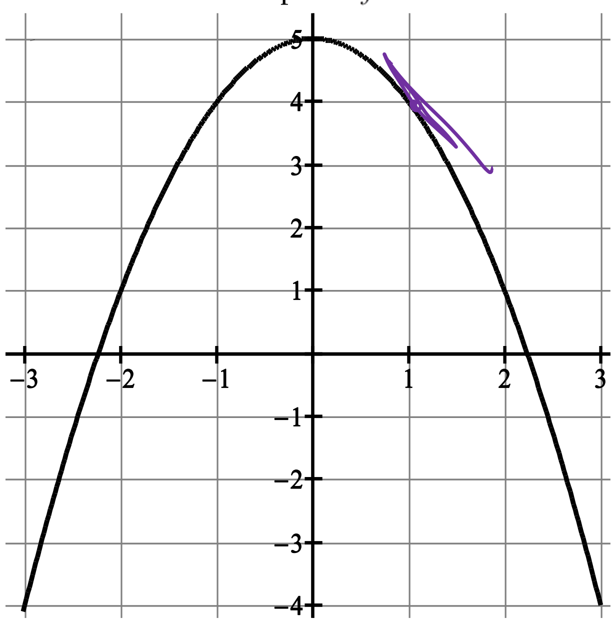
$j'(0) = \dfrac{g(0) \cdot f'(0) - f(0) \cdot g'(0)}{(g(0))^2} = \dfrac{-2 \cdot 2 - (-1)(1)}{(-2)^2}$

$h'(0) = g(0) \cdot f'(0) + f(0) \cdot g'(0)$
$-2(2) + -1 \cdot 1$

1 Suppose that $f(1) = 2$ and f' is the function shown below. Let $m(x) = x^3 \cdot f(x)$

$m'(x) = f(x) \cdot 3x^2 + x^3 f'(x)$

Graph of f'



a) Is f(x) increasing or decreasing at x = -3?

$f'(-3) = -4 \therefore f(x)$ is decreasing @ $x = -3$

b) Find the equation of the tangent line to f(x) at x = 1.

$f(1) = 2 \quad f'(1) = 4$

$y - 2 = 4(x - 1)$

c) Evaluate $m'(1)$

$m'(1) = f(1) \cdot 3 + 1 \cdot f'(1)$
$2 \cdot 3 + 1 \cdot 4 = 10$

d) Show that m is increasing at 2

$m'(2) = f(2) \cdot 3(2)^2 + 2^3 \cdot f'(2)$
$= f(2)(12) + 8 \cdot 1$
$+ (12) + 8 > 0 \therefore$ increasing

$f(2) > 0$
b/c $f(1) > 0$ and f is increasing since $f'(0)$ – $f'(2) > 0$

e) Estimate $f''(1)$

≈ -2